Measuring gas flow through narrow tubes

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First Year Laboratory Report

Oct 2014

Abstract

The first aim of the experiment was to determine the viscosity of two ideal gases; helium and argon, in a constant volume environment, through thin tube, and from this also determine the mean free paths and collision cross-sections. The second aim was to measure the flow rate of each gas under a low pressure environment.

The main results we obtained for helium were as follows;

Viscosity (η) = $(3.0 \pm 0.6) \times 10^{-5}$ Pas

Pumping speed (S) = $(8.0 \pm 0.3) \times 10^{-4}$ m^3s^{-1}

and for argon;

Viscosity (η) = $(2.5 \pm 0.5) \times 10^{-5}$ Pas

Pumping speed (S) = $(6.4 \pm 0.2) \times 10^{-4}$ m^3s^{-1}

1. Introduction

When fluids flow through a pipe, the way in which they flow is determined by many factors including the size of the molecules and a property of the fluid known as viscosity. There are two flow regimes known as laminar and turbulent flow. These are illustrated by Figure 1 below. In laminar flow the fluid flows smoothly in a parabolic profile due to friction (due to viscosity) with the pipe walls.

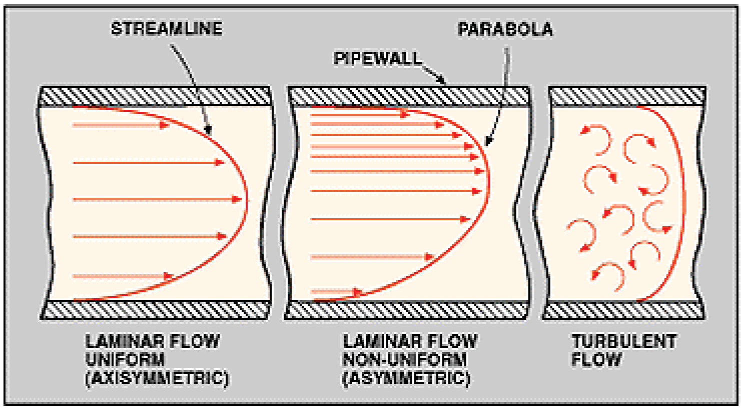

Fig 1. Schematic diagram for laminar flow [1]

While with turbulent flow, the motion of each layer (and on a molecular level) is far more random and chaotic. As part of our investigation into laminar flow we estimated the mean free path and collision cross-section of each gas. The mean free path of a gas is defined as the mean distance a gas molecule will travel after a collision (either with another molecule or the container). The collision cross section is related to the mean free path as a gas molecule will have a probability of colliding with another molecule that depends on the area of the gas molecule. The larger the collision cross section, the more likely a collision is to occur hence the lower the free mean path.

2. Theory

From the kinetic theory of gases the flow rate of a gas flowing in a pipe with one end at pressure p_1 and the other at a lower pressure p_0 is given by Poiseuille's equation. Where V_0 is the volume of the pipe system, a is the radius of the tube and l is the length of the tube.

$$Rate\ of\ flow\ (Q) = \frac{\pi a^4 (p_1{}^2 - p_0{}^2)}{16\eta l} = -V_0 \frac{dp_1}{dt} \quad (1).$$

It can be shown that by solving the differential Equation 1 by separation of variables, a linear relation can be derived as

$$\ln\left(\frac{p_1 - p_0}{p_1 + p_0}\right) = -\frac{\pi a^4 p_0 t}{8\eta l V_0} + a\ constant \quad (2)$$

Since Equation 2 is a straight line and all the variables can be determined in the laboratory, the gradient of $\ln\left(\frac{p_1 - p_0}{p_1 + p_0}\right)$ against time will be equal to the gradient, hence viscosity can be determined. Once the viscosity has been determined it is possible to calculate the mean free paths of each gas respectively. The equation relating viscosity with the mean free path of a gas is given as

$$\eta = \frac{nm\lambda}{3} \quad (3)$$

where n is the number density which is the number gas molecules per m^3, m is the mass of an individual gas atom and λ is the mean free path. Although n is not known explicitly it is possible to estimate it using our experimental values with the ideal gas equation given as;

$$P_0 V_0 = n_m RT \quad (4)$$

where n_m is the number of moles of the gas, R is the Boltzmann constant converted when dealing with molar quantities and $T$is the temperature of the gas.

Once n_m has been determined the number density can be calculated using the following relation of;

$$n = n_m \frac{N_A}{m} \rho \quad (5)$$

where ρ is the density of the gas and N_A is Avogadro's number (defined as the number of carbon 12 atoms in 12g of Carbon-12).

Another flow regime which occurs at low pressures is molecular flow. Because of this low pressure, the mean free path of the gas atoms is larger than that of laminar flow, so collisions with the tube walls are more common than those with other gas atoms.

$$rate\ of\ flow\ (Q) = -V_0 \frac{dp_1}{dt} + Sp_r \quad (6)$$

where p_r is the limiting residual pressure which arises from leaks and other minor sources which mean that the pressure in the pipe system will not reach zero. Again by solving the differential equation, the pumping speed S can be found.
Hence the following linear relationship is derived,

$$\ln(P_1 - P_r) = \frac{-St}{V_0} + a\ constant.$$

3. Experimental Method

The main apparatus given to us is shown in Figure 2 below.

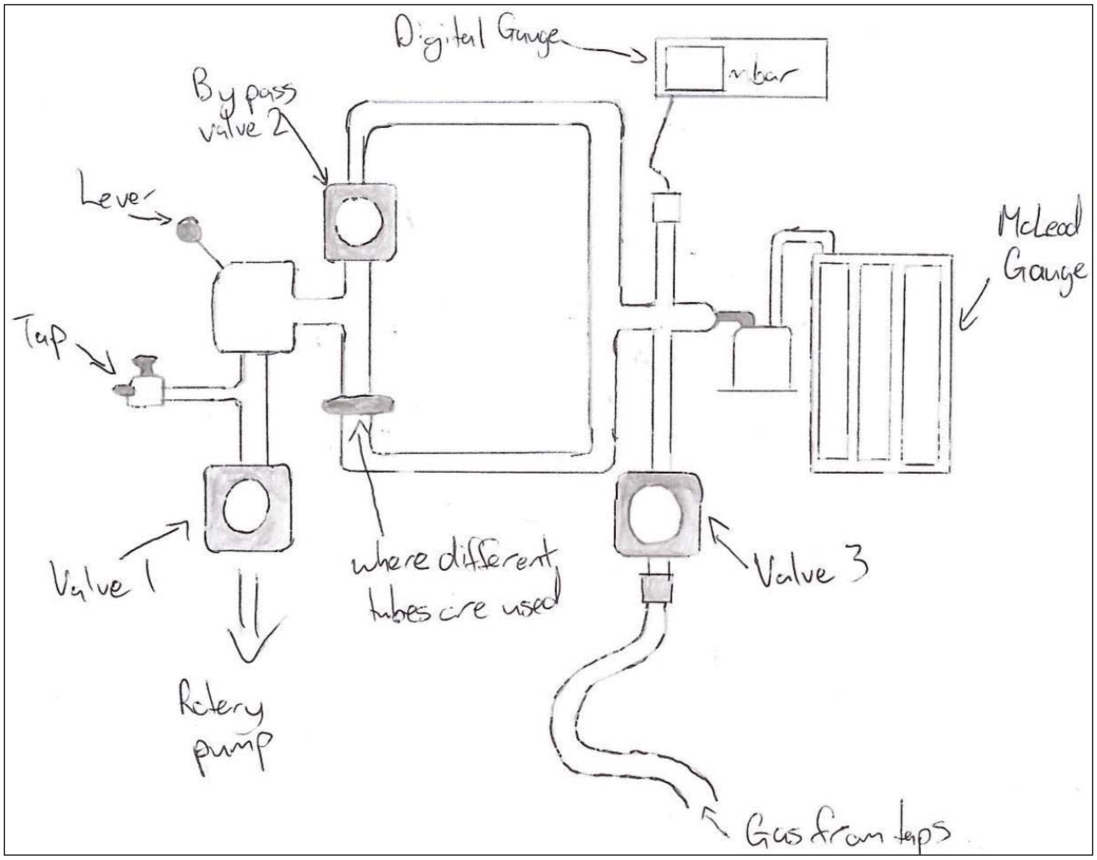

Fig 2. Diagram of the apparatus used for measuring gas flow.

We were also given 2 tubes. The longer was of length (3.00 ± 0.05) cm (giving a percentage uncertainty of 1.67%) and diameter (0.200 ± 0.005) mm (giving a percentage uncertainty of 2.5%). The measurements for length were made using calipers and those for diameter were made using a Vernier scale on a travelling microscope. The uncertainties were determined to be half the smallest reading on the particular scale. The dimensions of the shorter tube were not included in the calculations for molecular flow therefore they were not measured.

From the theoretical results established in the section 2 we knew a linear relationship with variables which were known. However we had to determine the volume of the pipe system as this was not known. To do this we were given a tube of known volume, and connecting it to the pipe system and evacuating the pipe system while the tube was kept isolated from the system.

Fig 3. Pipe system as used during the experiment.

Then the valve was opened allowing the air in the tube to diffuse through the system. To determine the volume we assumed that the air behaved as an ideal gas. Hence using the gas law

$$P_1V_1 = (V_1 + V_2)P_2 \quad (7)$$

where P_1, V_1 *and* P_2 are known, the volume was estimated.

3.1 Experimental procedure

To investigate laminar flow, a relatively long tube was used. After evacuating the entire system and then filling it with the gas with which we were using for experimenting (either helium or argon). When the valve was opened, the gas flowed through the U bend causing a drop in pressure which was measured with respect to time for a time of 5 minutes in 10 second intervals. Care was taken to try and avoid parallax error, so measurements were made at eye-level.

For molecular flow low pressures were required, so to make sure that the only gas present in the pipes was the gas we were measuring, the entire system was evacuated and then the gas was pumped in. A shorter tube compared with that used in laminar flow was used during the experiment. The whole system was then evacuated again until it reached the appropriate pressure (approximately 10mbar) according to the pressure gauge. Taking an initial reading on the McLeod Gauge and then opening the valve allowing the gas molecules to flow through the system. Measurements were made every minute until the pressure was measured to be the same after a consecutive minute. This value was also the residual pressure.

4. Results

Presented below are graphs for the laminar and molecular flows of each of helium and argon. They were produced using the Matlab Least Squares Fit package [2].

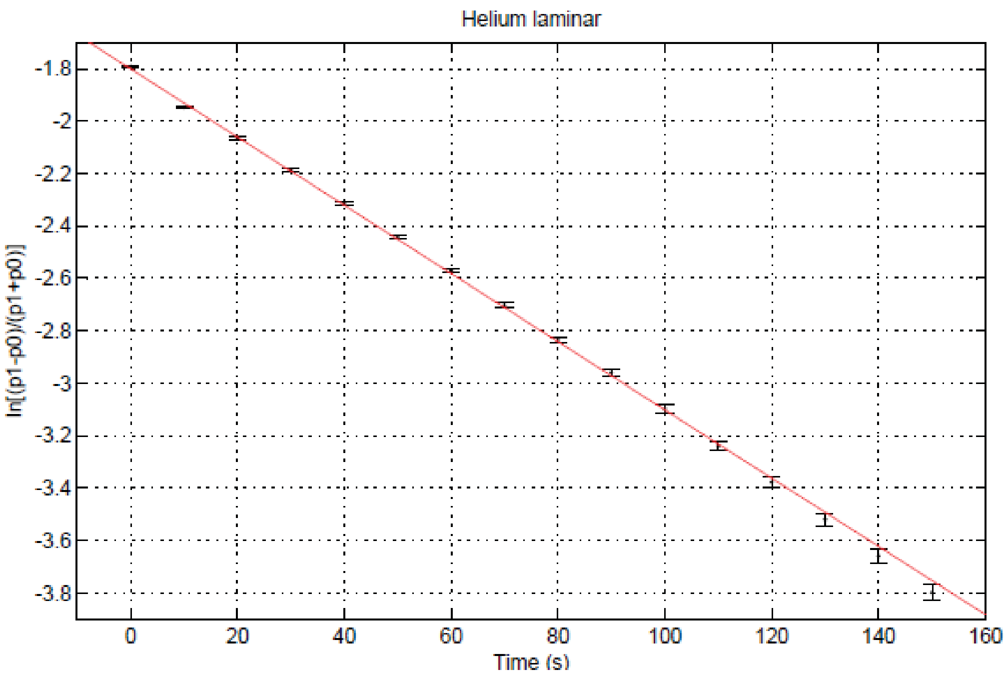

Fig 4. The final graph used in the calculations for laminar flow of helium. Data points were removed until the minimum value for the reduced-χ^2 (2.51) was obtained. This was done since Equation 2 for laminar flow was only valid for the linear part of the graph. At t=160s the data points started to curve away from the straight line, meaning the turbulent flow was taking place. From the value for the reduced-χ^2 we can say that the goodness of fit is close to the accepted value ($0.5<\chi^2_{red}<2$).

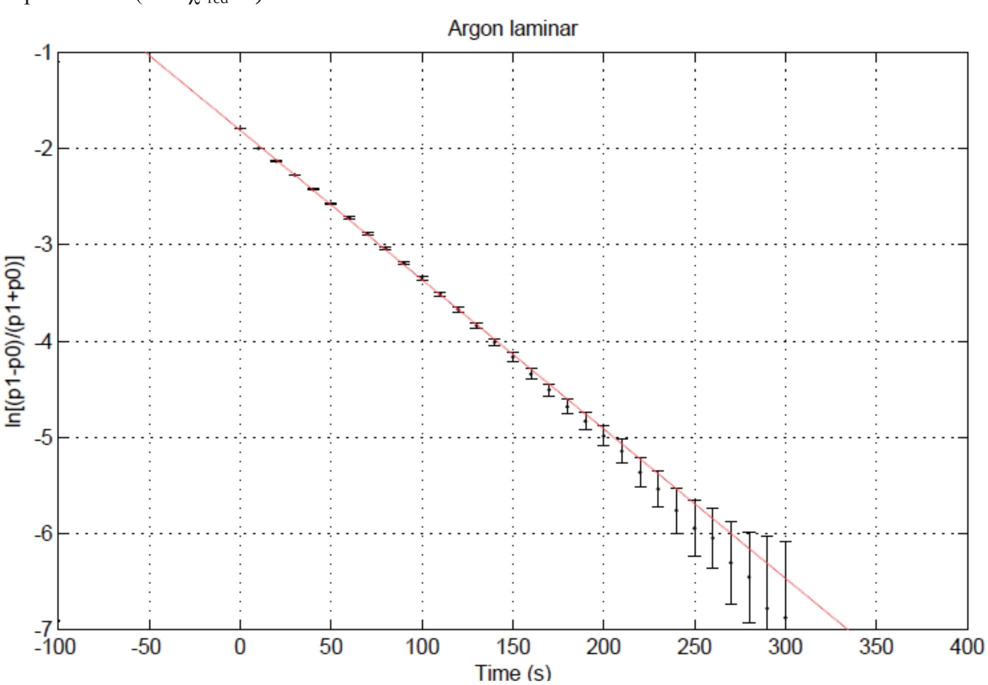

Fig 5. Similar to Fig 4, but for argon, rather than helium. In this instance, removing data points did not improve the reduced-χ^2 therefore all points were kept. This was because, for argon more so than helium, the data points after at time of about 160s did not deviate as much from the line of best fit. Also, the error bars at later times, still passed through the line, whereas with helium they did not. The reduced-χ^2 for this graph was 3.46 (to 2 decimal places) therefore a linear fit was not so appropriate for this data.

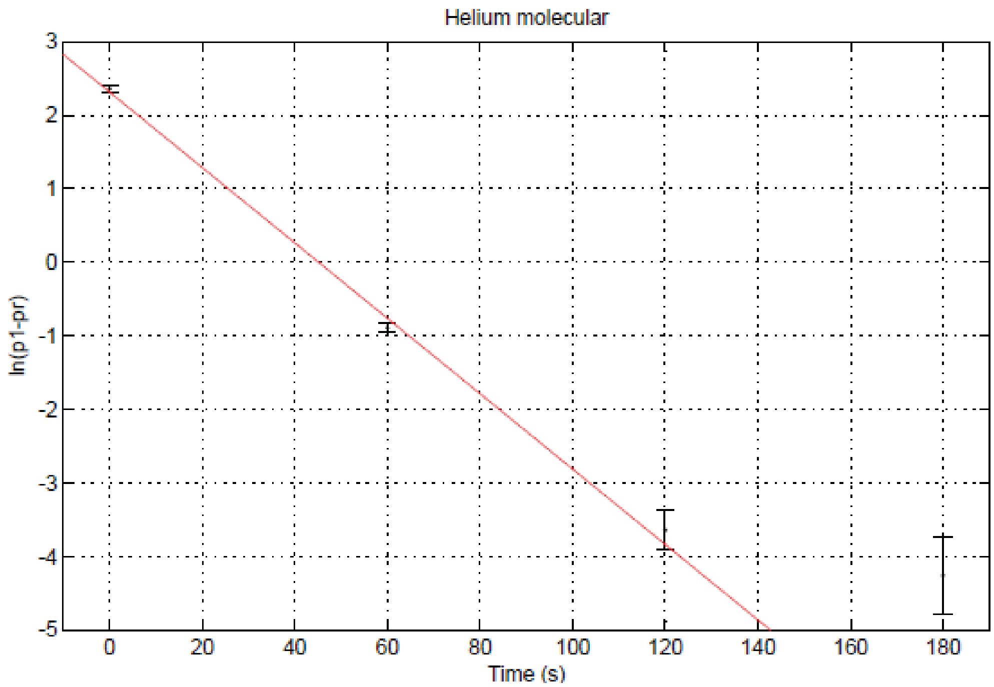

Fig 6. The final graph for the molecular flow of helium. A calculation for the reduced-χ^2 gave 15.5, suggesting a very bad fit. This can be seen from the final error bar being far from the line of best fit. However, the data point could not be removed as there was no physical interpretation as to why this would be appropriate.

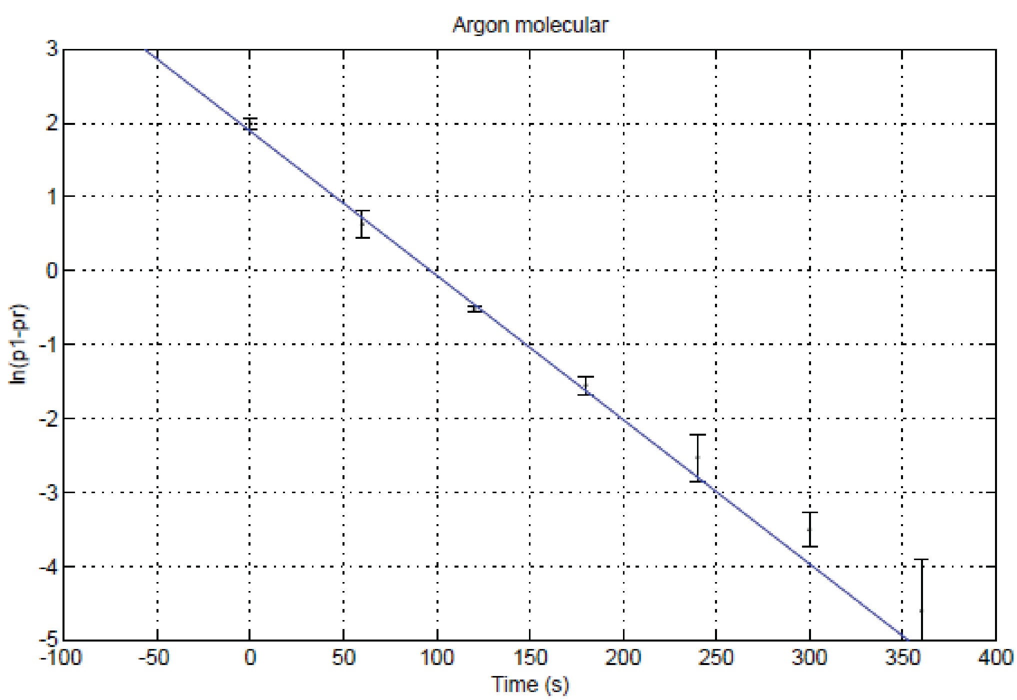

Fig 7. Similar to Fig 6 but for argon. This time it took a longer time to reach the limiting residual pressure and the error bars were closer to the line of best fit. This gave a better goodness of fit with a reduced-χ^2 of 2.03 (close to the definition of a good fit with a reduced-χ^2 of less than 2)

Using the gradients (and intercepts in the case of molecular flow) of these graphs from the Matlab package the corresponding viscosities and pumping speeds were calculated. The final formulas used for the errors on viscosity and pumping speed respectively were;

$$\sigma_\eta = \frac{\pi a^3}{2mlV_0}\sqrt{(p_0\sigma_a)^2 + (\frac{a\sigma_{p_0}}{4})^2 + (\frac{ap_0\sigma_m}{4m})^2 + (\frac{ap_0\sigma_l}{4l})^2 + (\frac{ap_0\sigma_{V_0}}{4V_0})^2} \quad (8)$$

$$\sigma_S = \sqrt{((c-m)\sigma_{V_0})^2 + (V_0\sigma_c)^2 + (V_0\sigma_m)^2} \quad (9)$$

where *m* is the particular gradient.

The main factor contributing to the uncertainty in viscosity was the fractional uncertainty in the radius (twice the fractional uncertainty in the diameter) and this lead to a percentage uncertainty of 20% (for both helium and argon) in the viscosity. On the other hand, the biggest contributor to the to the pumping speed of helium was the intercept of the graph (percentage uncertainty of 3.55%), leading to a final percentage uncertainty of 3.67% in the pumping speed. For argon the biggest contributor was the gradient of the graph (with percentage uncertainty 2.60%), giving a final pumping speed percentage uncertainty 3.02%.

As the lab script [3] only specified to estimate the mean free paths and collision cross-section the rest of the variables in Equations 3,4 and 5 were treated as constants. This meant that the fractional uncertainties in viscosities were the same as those for mean free path and collision cross-section.

The fractional uncertainty in the Reynolds number could be calculated through combining the fractional errors in radius and viscosity in quadrature. This gave percentage uncertainties of 20.7% in Reynolds number for both helium and argon.

5. Discussion

After analysing the data from our experimental results, we have found that the viscosity of helium is greater than that of argon, which is in disagreement with accepted values which suggest the opposite. This also means that the calculated mean free path and collision cross section of helium are also significantly larger than the true value. A reason for such discrepancies is due to the high level of fractional uncertainty in measuring the radius of the tube for laminar flow and that there are a 4^{th} power involved in the equations causing a larger uncertainty in the viscosity. Additionally, the true volume of the pipe system was not measured directly and the value of the volume used is an average from other groups conducting the experiment. Since each apparatus was different, this could potentially have meant that the volume of our system could vary significantly compared with the volume we have used in our calculations. However we assumed that this was the true value of our system because it would have been difficult to estimate the true error from data from other groups. Having said this, the uncertainty on our volume was determined in the usual way through partial differentiation and combining in quadrature. This lead to the equation;

$$\sigma_{V_0} = \frac{V_1 \sigma_p}{p_2} \sqrt{1 + (\frac{p_0}{p_2})^2} \quad (6).$$

The percentage uncertainty in V_0 was 1.2%.

6. Summary

The experimental values of viscosity, mean free path, collision cross-section, Reynolds number and pumping speed are presented below in Table 1.

	Helium	Argon
Viscosity (η)	$(3.0 \pm 0.6) \times 10^{-5}$ Pas	$(2.5 \pm 0.3) \times 10^{-5}$ Pas
Pumping speed (S)	$(8.0 \pm 0.3) \times 10^{-4}\ m^3 s^{-1}$	$(6.4 \pm 0.2) \times 10^{-4} m^3 s^{-1}$
Collision Cross-section (σ)	$(9 \pm 2) \times 10^{-20}$ m^2	$(3.5 \pm 0.7) \times 10^{-20}$ m^2
Mean Free Path (λ)	$(4.4 \pm 0.9) \times 10^{-8}$ m	$(1.2 \pm 0.2) \times 10^{-8}$ m
Reynolds Number (R)	$(1.4 \pm 0.3) \times 10^{3}$	$(5 \pm 1) \times 10^{3}$ m

Table 1. Summary of all the experimental results.

All uncertainty values in Table 1 are quoted to 1 significant figure and the value is then quoted to the appropriate significant figure following this.

7. **References**

[1] http://www.omega.com/techref/flowcontrol.html 15:23 10/11/2014

[2] Matlab Least Squares Fit, lsfr26.m available from Teachweb, http://teachweb.ph.man.ac.uk/.

[3] Lab script for First year lab experiment on gas flow through narrow tubes, I. Grant, pp. 2

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