Heights of lunar mountains

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Abstract

Through measurement of the shadow lengths of lunar features from an image obtained at Pic du-Midi, heights of various lunar features were calculated. The heights of eight measured features ranged between 0.14 ± 0.01 km and 2.62 ± 0.07 km. The main uncertainties in the heights obtained, were due to a systematic error and the resolution of the telescope. This systematic error was caused by not measuring the length of the shadows at the 50% intensity location.

1. Introduction

Observations of the moon have been carried out for centuries. During the 1960's several observations of the moon were taken during half-moon and close to half-moon from the observatory at the Pic-Du-Midi in the south of France. The telescope at the observatory is a refracting telescope with a single achromatic lens of diameter 60cm [1]. One of these images was examined under a travelling microscope in order to make estimates of the heights of various features such as craters and mountains on the moon. The image used for this experiment was from an image developed on a photographic plate, which is a relatively old technology but provided sufficient detail to make measurements.

2. Theory

In order to calculate the height of a lunar feature, several pieces of data are required. For all of the geometry used here, an assumption that the moon was a perfect sphere was made. Using images of the moon close to or at half-moon, the distance to the terminator, *L*, (the dividing line between the dark and light side of the moon) to the feature can be made. Additionally, the shadow length, *s*, can be estimated from the differences in brightness on the image. With known data about the moon, including its radius, *R*, and using simple geometry for the case of a half moon we obtain the situation as shown in Figure 1.

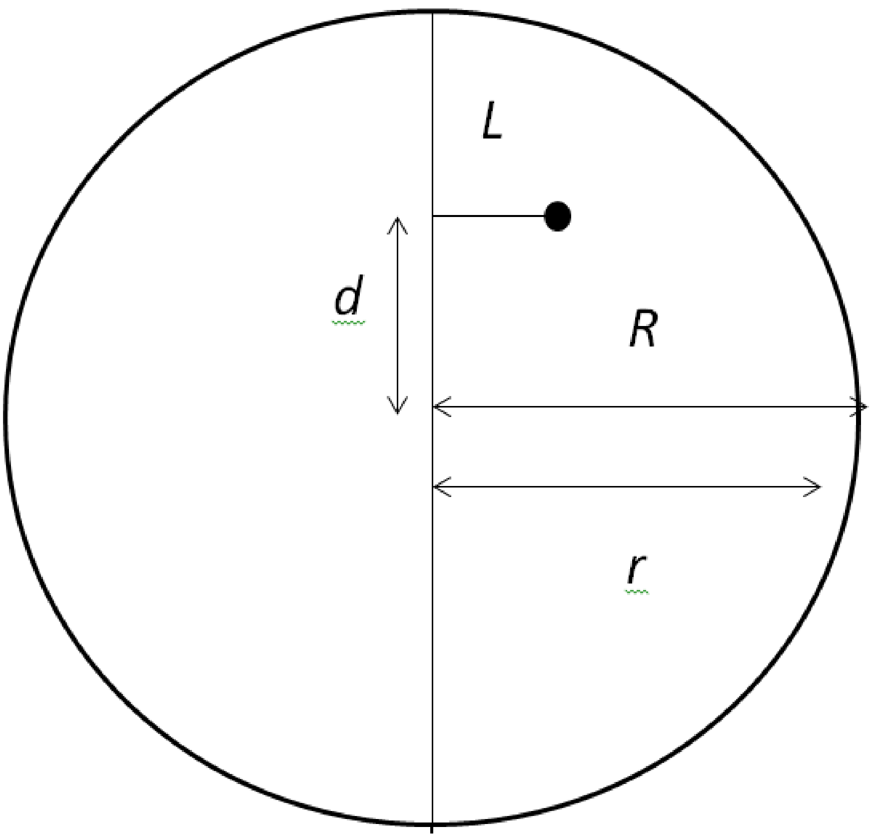

Fig 1. Diagram showing quantities measured from an image of the moon during the half-moon

The situation in Figure 1 is for a cross-section through the center of the moon. Projecting these quantities for a cross-section through the location of the feature we obtain the case as shown in Figure 2.

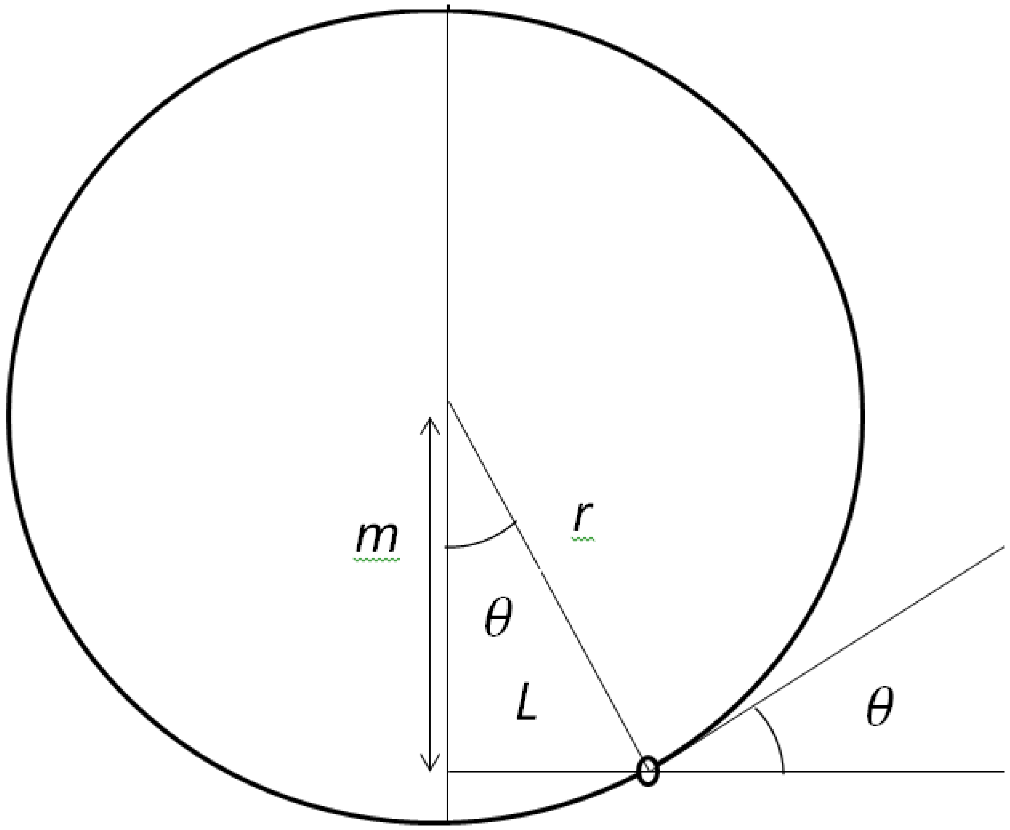

Fig 2. Diagram of a projection of Figure 1onto a cross-section through the feature.

Since we have projected it onto the circumference of the cross-section, there is a new vertical distance from the center to the projection, *m*, but the distance from the terminator remains the same. The sunlight is tangential to the circumference of the cross-section, subtending an angle of illumination, θ. Figure 3 shows the relation between the height of the feature, *h*, *s* and θ.

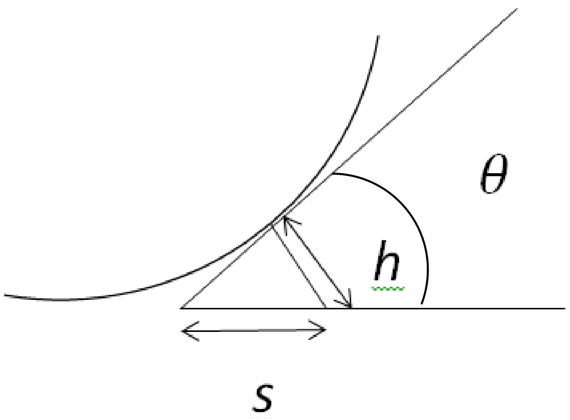

Fig 3. Diagram of the situation at the point where the tangent of the Sun's rays meet the cross-section.

Using similar triangles, the height of the feature is given by

$$h = \frac{sL}{r}, \tag{1}$$

where *r* is the radius of the circular cross-section where the feature lies and is given by

$$r = \sqrt{R^2 - d^2}, \tag{2}$$

where *R* is the radius of the moon and *d* is the vertical distance from the center of the moon to the feature. These relations hold when the terminator is not a line but is an ellipse, and so has the equation

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1, \tag{3}$$

where *x* and *y* are the horizontal and vertical positions taken from the center of the image and taking the semi-major axis, *b*, as the radius of the moon, a value of the semi-minor axis, *a*, was estimated and the equation of the terminator was found. Once a value for *a* was obtained, a value of *L* can be found from measurements on the image itself and a value of *h* calculated.

3. Experimental procedure and set-up

The image used was similar to Figure 4. It was a negative image, meaning that the dark area is the illuminated part, and the light area is the dark part of the moon. This means that sunlight was coming from the right in Figure 4.

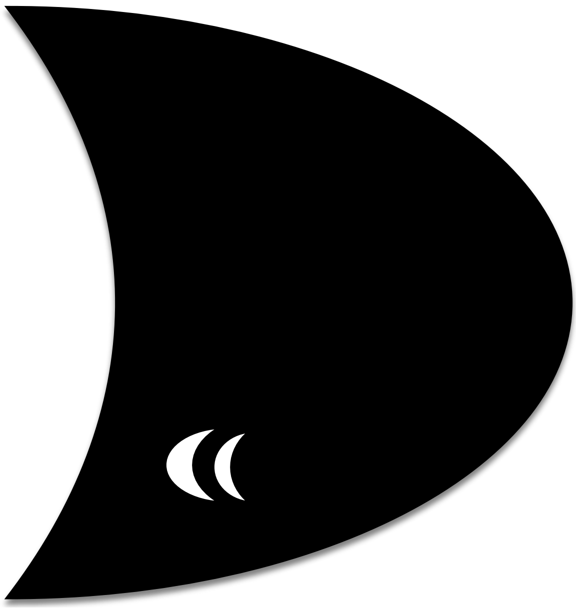

Fig 4. A simple diagram of a negative image of the moon, with an example of the shadows cast by a feature in white. The dark area on the figure is the illuminated part of the moon, whereas the light part of the figure corresponds to the dark part of the moon.

Figure 4 shows exaggerated shadows cast by a feature on the Moon. First of all, the travelling microscope is moved down from the top of the image to the vertical position of the feature, by winding it mechanically or by pressing a button on the travelling microscope. When the feature is seen in an eyepiece on the travelling microscope, the microscope can then be focused to see the shadows in more detail. By then moving the travelling microscope from left to right, position measurements for the starts and ends of the left and right shadows are taken by reading a Vernier scale which is built into the travelling microscope, for horizontal and vertical measurements. The start or end of a shadow is not definite; therefore the best method for determining a position is to estimate the 50% intensity level, as illustrated in Figure 5. In order to calculate the height of the feature in km, a conversion factor from mm to km was required. This was found by using a known value for the diameter of the moon in km [2] and then measuring the diameter of the image under the microscope. Dividing these two quantities gave a conversion factor in km/mm.

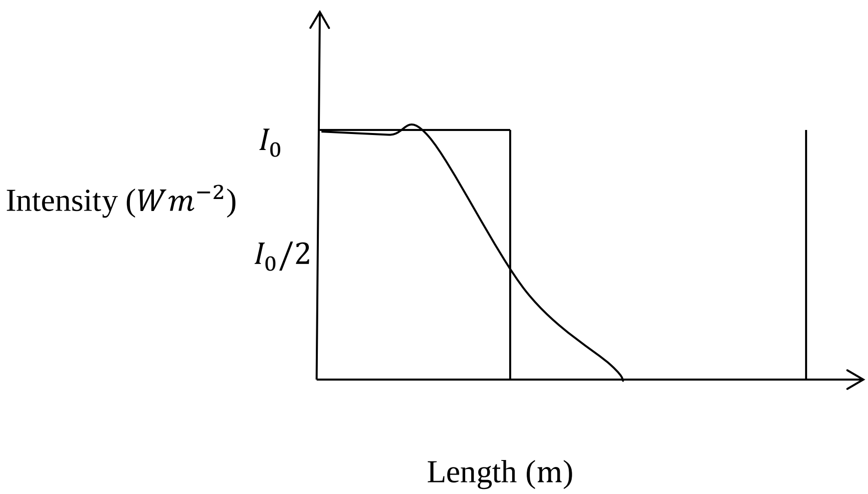

Fig 5. A diagram showing the effect of diffraction on the intensity of light observed by an optical instrument, shown by the curved line. The true intensity is represented by I_0. The most accurate place to estimate the true shadow length is at the midpoint of the intensity drop.

4. Results

After measuring shadow lengths for 8 features an estimate for the height of the feature was made using Equation 1. Values for *h* are shown in Table 1.

Feature	*h* (km)
A	2.41 ± 0.10
B	1.11 ± 0.13
C	0.81 ± 0.04
D	1.54 ± 0.08
E	0.53 ± 0.11
F	2.62 ± 0.07
G	0.14 ± 0.01
H	0.27 ± 0.06

Table 1. Table showing the heights of each feature measured from A to H, along with their uncertainties.

It was found that features D and F were mountains, A, B, C and H were craters with their interiors lower than the surrounding plains, whilst E and G were craters with their interiors higher than the surrounding plains. A feature was classified as a mountain if no right shadow was seen. To determine whether the interior of a crater was above or below the surrounding plains, the length of the left and right shadows were compared. If the length of the left shadow was longer than the right, it was concluded that the crater had an interior higher than the surrounding plains and vice versa for a crater with an interior lower than the surrounding plains. Both of these cases are illustrated in Figure 6.

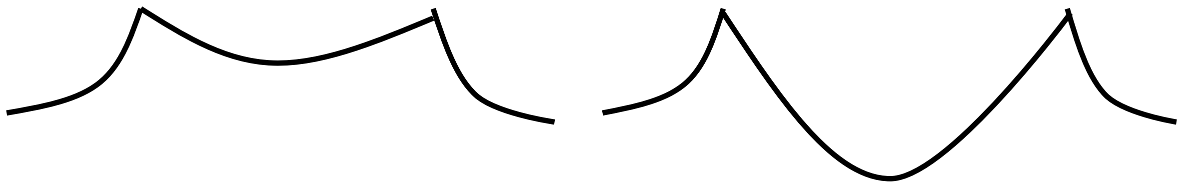

Fig 6. Simple diagrams of a crater with an interior higher than the surrounding plains on the left and a crater with an interior lower than the surrounding plains on the right.

5. Discussion

The error in the heights of the lunar features varied significantly from approximately 3% to 24%. A small contribution to these uncertainties arose from the precision of the travelling microscope. The travelling microscope was capable of measuring to the nearest thousandth of an mm, suggesting a percentage error of approximately 10^{-3}%. There was error in finding the value of a, which arose partly from the error of the travelling microscope. The error in a propagated through values of L and r, but again due to the error mainly arising from the travelling microscope, this was very small. The moons diameter was taken to be 3474 ± 1km and a total error was found by adding the error of the travelling microscope and resolution in quadrature. The main contribution to uncertainty in height arose from two main sources. Firstly that the shadow lengths of features were not measured at points where the intensity was at 50% but at the point where it was perceived that the shadow ended. This suggested, as shown in Figure 5, that the lengths of our shadows were underestimated and hence the heights of the lunar features were smaller, as suggested by Equation 1. Despite not doing this, an error arising from the resolution of telescope was taken into account. From consulting a textbook, the best resolution possible for the telescope was $0.4''$ [3]. Using this value, an estimated value for the fractional uncertainty in resolution, σ_{res}, was given by

$$\sigma_{res} = \frac{0.4''}{0.5' \times 60} = 0.0222, \tag{4}$$

with the denominator being the angular size of the moon in arc-seconds. This affected the errors on the conversion factor and the accuracy of the shadow lengths. Taking the resolution into account, the error in measuring the diameter of the moon under the microscope was ± 0.04 mm. This error is much greater than the error in the precision of the microscope. This was our second major source of error.

6. Conclusions

Despite the major error of not measuring the shadow length at the 50% intensity level, the ability to confidently illustrate the approximate structure of crater features was achieved. If a repetition of the experiment were carried out, the measurements of the shadow lengths would be done at the 50% intensity level. However, since these measurements are done by eye, it is subjective. Therefore, two measurements would be taken by different people in order to get a range where the 50% intensity lies and hence an improved result for shadow length. A further improvement would be to use several photographs of the moon with different terminators (such as at half-moon) in order to get more values for the heights of the same feature, and then check for consistency.

References

[1] Rackham, T, “Installing a Big Telescope for Moon Research”, *New Scientist*, Volume 13, No. 274, page 366, 1962.

[2] http://nssdc.gsfc.nasa.gov/planetary/factsheet/moonfact.html, *Moon Fact Sheet*, NASA, (16/12/2015).

[3] Dragesco, J, *High Resolution Astrophotography*, Cambridge University Press, 1st Ed, 1995, page.17.

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