The Boltzmann constant

John Cobbledick and Meirin Evans

9437432 and 9214122

School of Physics and Astronomy

The University of Manchester

First Year Laboratory Report

Jan 2015

Abstract

Through measurement of the current and voltage through a semiconductor diode using a voltmeter and multimeter, measurements of the ratio of the elementary charge to the Boltzmann constant were made at the freezing point and boiling point of water. From this data an estimate of absolute zero was made. The result for the ratio of elementary charge to the Boltzmann constant was 10848 ± 11 KV^{-1}, whilst for absolute zero a value of -264.8 ± 1.2 ^{0}C was obtained.

1. Introduction and Theory

The Boltzmann constant is not a fundamental constant of nature such as the elementary unit of charge or Planck's constant. It can be thought as a conversion factor between how we relate temperature and the kinetic energy of molecules or in more general terms, heat. A prime example of this is for the average kinetic energy of an ideal gas molecule given by $\frac{3}{2}kT$ where T is the temperature in kelvin of the gas. Another example of its use, which was the basis of the experiment, is the movement of electrons in a semiconductor. The electrons' movement is very similar to molecules of gas. A semiconductor diode consists of two pieces of semiconducting materials which have different electron densities and when brought together electrons will transfer from the material with the highest electron density to the lower. This therefore produces a current and a voltage. The contact voltage (V_C) is proportional to $e^{-\frac{eV_C}{kT}}$. This arises due to the fact work must done on an electron to move it to the other side, which is given by QV, where Q is the charge of the electron and V is the voltage, and that the probability distribution of the kinetic energy of molecules is given by $e^{-\frac{E}{kT}}$ where E is the energy of any given molecule at a temperature T [1].

2. Experimental procedure and set-up

Below in Figure 1 is an apparatus diagram and in Figure 2 a more detailed circuit diagram which was used for the experiment at both temperatures, with the diode being placed inside a container with water heated at 100°C and a thermometer for the data at high temperatures.

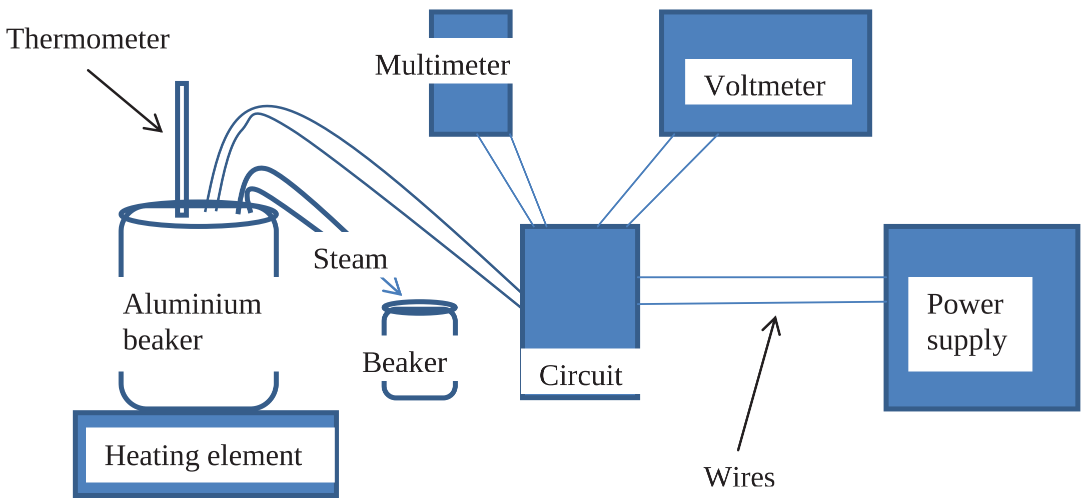

Fig 1. A simple diagram of the apparatus used during the experiment.

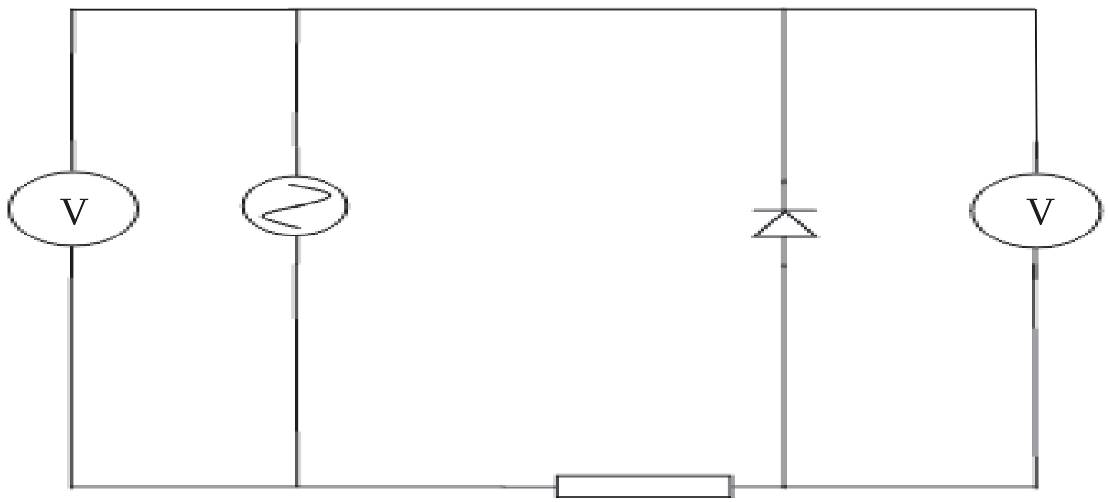

Fig 2. A Circuit diagram of the apparatus used during the experiment. The supply voltage was measured using a multimeter set to measure voltage and the diode was within a copper block that could be placed in a beaker of boiling water or ice.

By using a resistor of known resistance, the voltage across the resistor was found by subtracting the voltage from the diode from the voltage across the power supply. This method was used rather than directly measuring the current because it fluctuated rapidly. Knowing the voltage across the resistor allows the current to be calculated through Ohm's law $V = IR$. Then a graph of diode voltage against log of current can be plotted. This procedure is repeated with the same resistor but with the diode immersed in a beaker of ice rather than boiling water to produce another graph. The two different sets of measurements are repeated with two other resistors. The values of resistors used in this experiment were 10kΩ, 100kΩ and 1MΩ.

3. Results

Presented below are graphs of diode voltage against log of current through the resistor for different resistors with the diode at different temperatures, produced using the Matlab Least Squares Fit package [2] Figures 3 to 8 are plotted for 29 degrees of freedom.

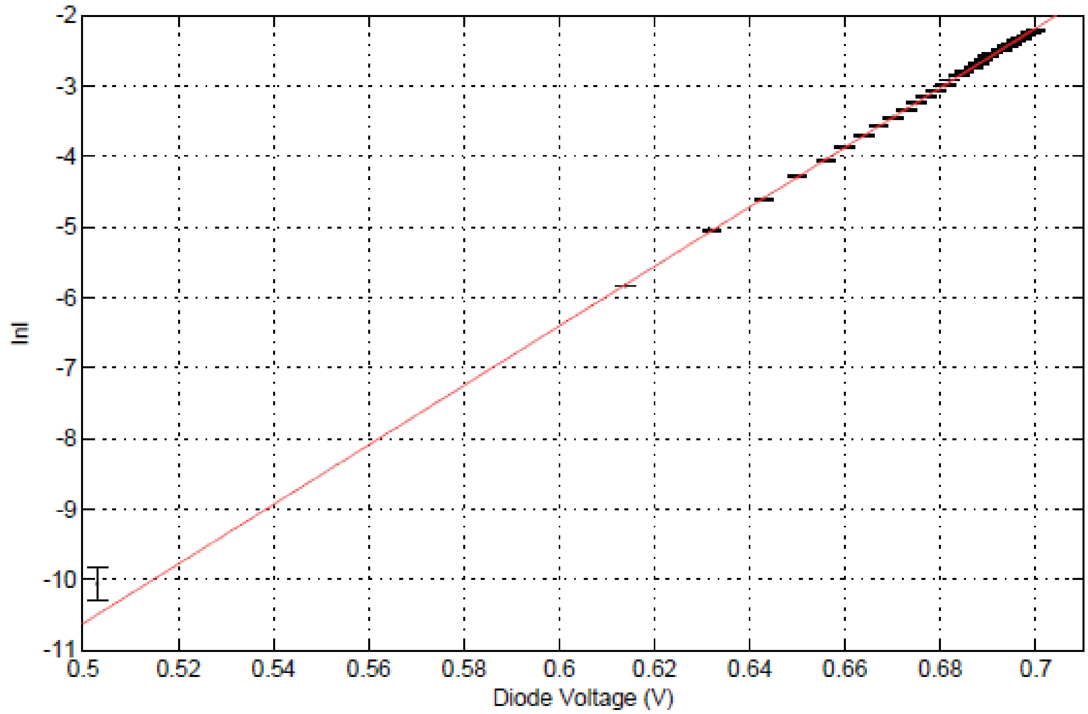

Fig 3. A plot of diode voltage against log of current using a 10kΩ resistor with the diode at 0^0C.

The reduced-χ^2 obtained using the package with the data from Figure 3 was 73.79, giving a very bad fit since this is far away from the definition of a good fit having a reduced-χ^2 of 2. This is a result of the error bars in all but the first data point being very small.

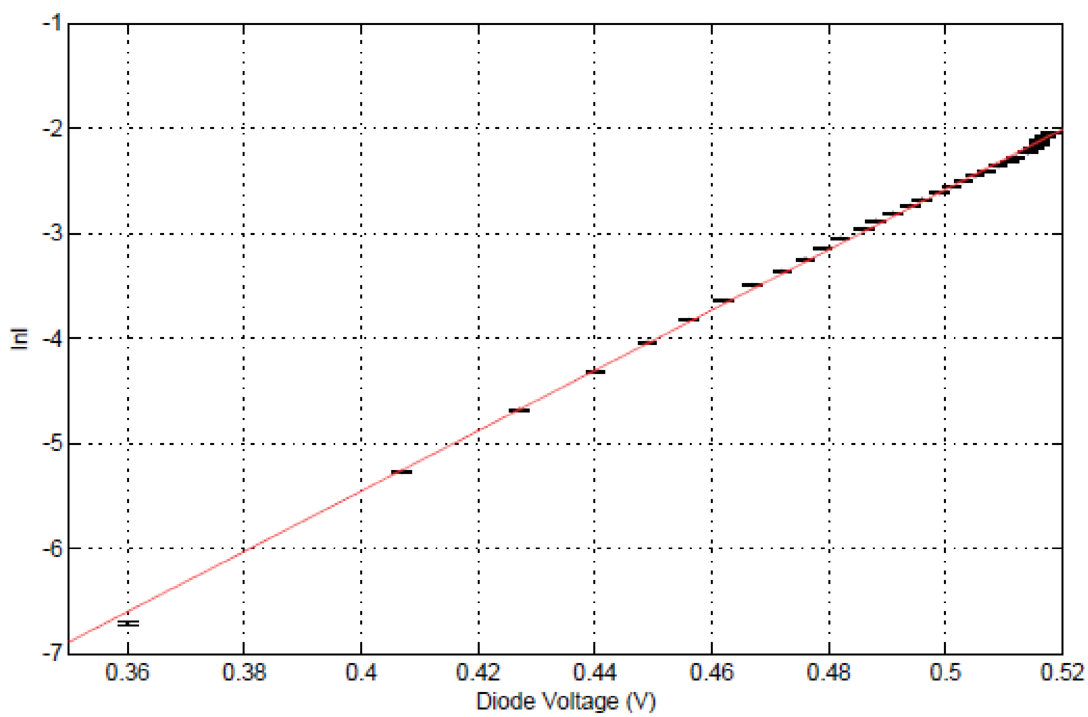

Fig 4. Similar to Figure 3 but for the 10kΩ resistor and the diode at 103^0C.

The number quoted for the temperature of water (in which the diode was immersed) in Figure 4 is the reading from the thermometer. Even though the water temperature could not surpass 100^0C it is possible that the thermometer was touching or close to the bottom of the metal beaker which was being heated by an electric heater, this would make the measured temperature more than that of the water. However, the heater did have to be kept on throughout the measurements to ensure that the water stayed at 100^0C. The reduced-χ^2 obtained with this data from Figure 4 was 299.9, giving an even worse fit than Figure 3. In this instance, as well as the small error bars for the data points towards the top of the graph, the first data point also has a small error bar compared to that of the first data point in Figure 3, leading to this higher reduced-χ^2.

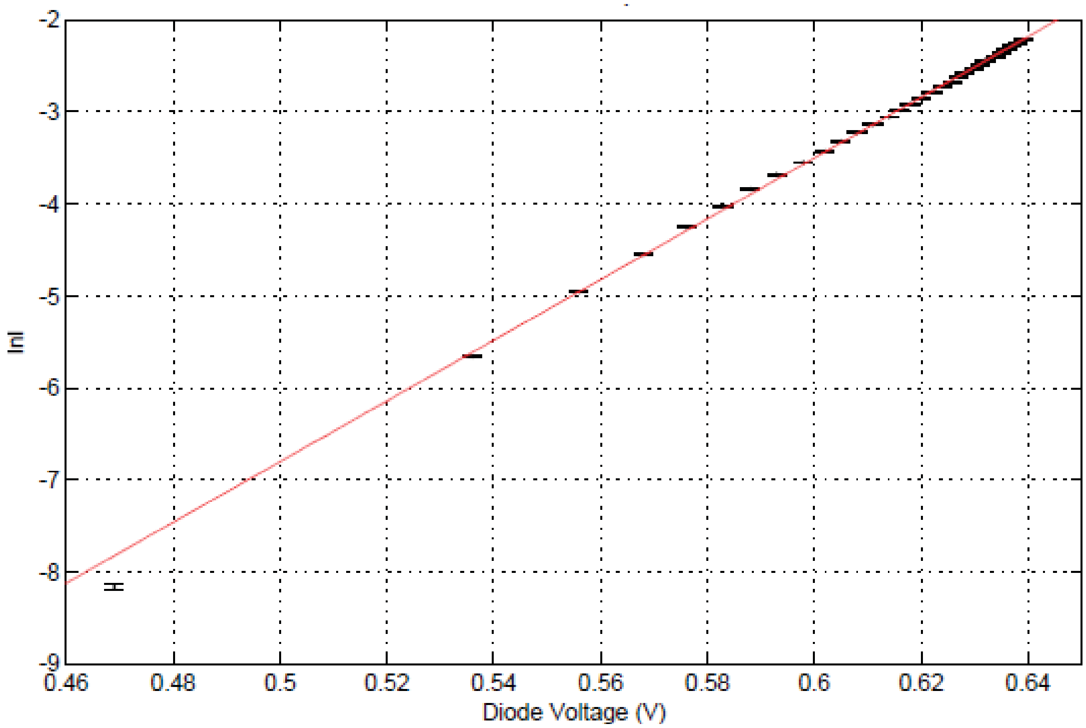

Fig 5. Similar to Figure 4 but for the 100kΩ resistor and the diode at 1^0C.

In Figure 5 the thermometer may have measured a temperature above 0^0C due to the ice within the beaker starting to melt. The ice had to be carried over from an ice machine to the apparatus and it took some time to start taking measurements after this. The calculation for the reduced-χ^2 from the package gave 213.1. This data gave a bad fit for the same reasons as for Figure 4. As well as this, the first data point is quite far from the line of best fit. A lower reduced-χ^2 than Figure 4 means that the data points towards the top of the graph must be closer to the line of best fit than in Figure 4.

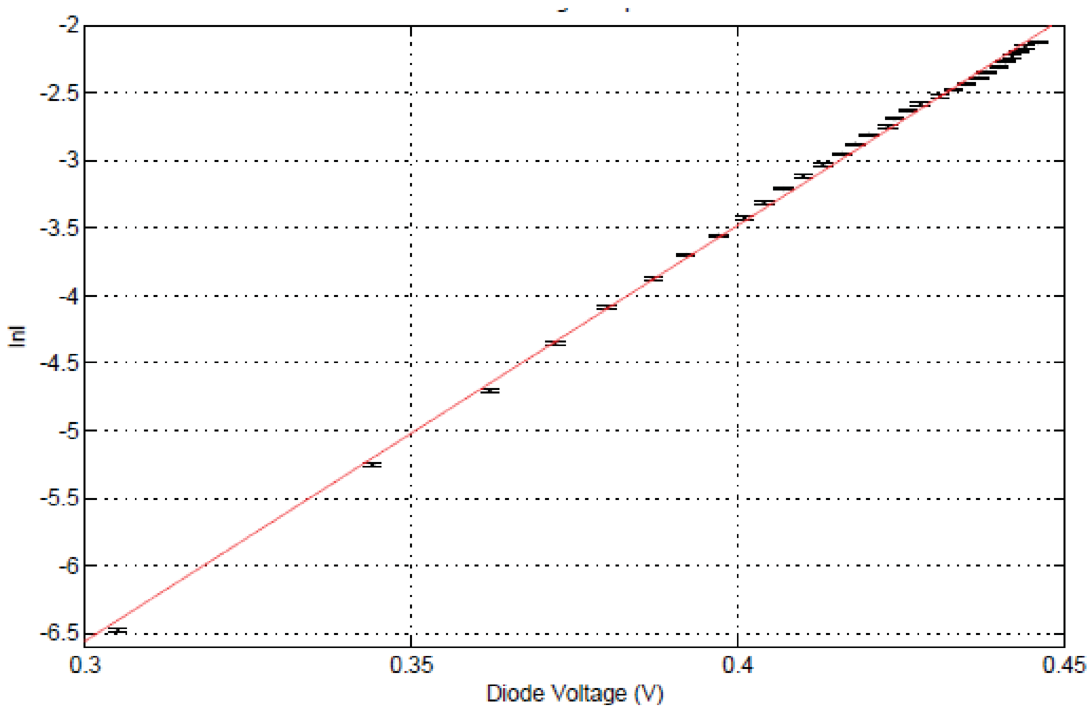

Fig 6. Similar to Figure 5 but for the 100kΩ resistor and the diode at 101^0C.

The temperature quoted in Figure 6 may be over 100^0C for the same reason as for Figure 4. The reduced-χ^2 produced for Figure 6 was 518.7. The reason this data gave a worse fit than Figures 4 and 5 is the data points fluctuating from the line of best fit towards the top of the graph (as well as those reasons already mentioned for Figures 4 and 5).

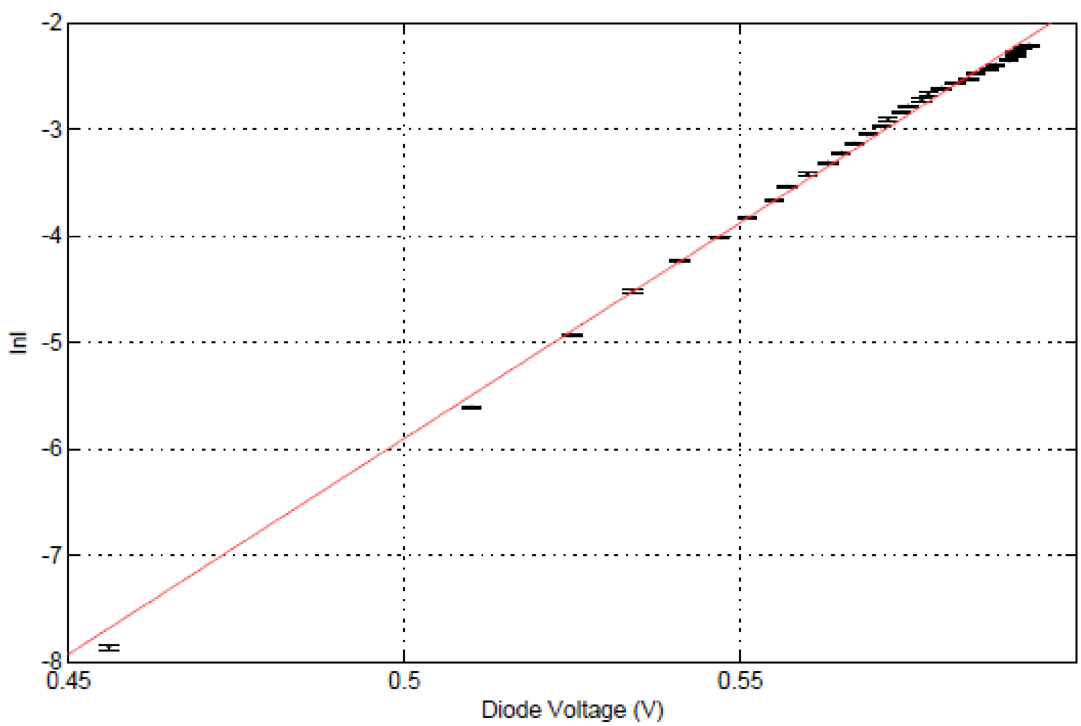

Fig 7. Similar to Figure 6 but for the 1MΩ resistor and the diode at 1^0C.

The quoted temperature in Figure 7 may be higher than the freezing point of water for the same reason as that of Figure 5. The reduced-χ^2 given for Figure 7 was 1043. This is due to the extra significance of data points towards the middle of the graph having very small error bars yet still being away from the line of best fit.

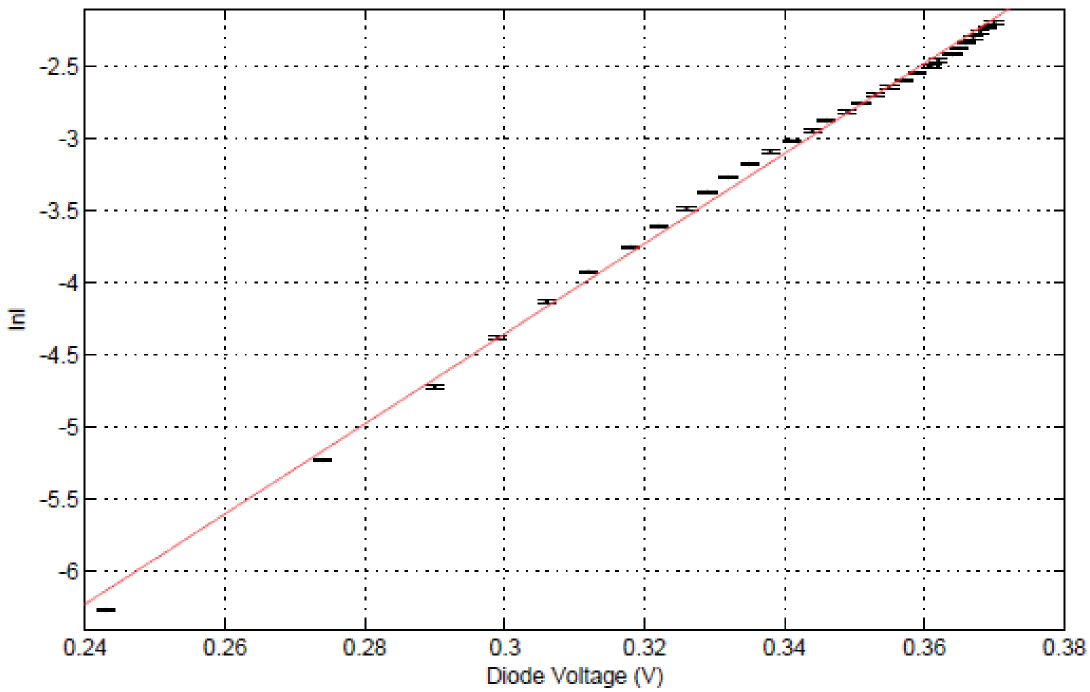

Fig 8. Similar to Figure 7 but for the 1MΩ resistor and the diode at 102^{0}C.

The temperature quoted in Figure 8 may also be over the boiling point of water for the same reason as for Figure 4. A reduced-χ^2 of 813.5 was produced for Figure 8. This is a bad fit for the same reasons as Figure 7 but slightly lower either because the data points towards the middle of the graph have bigger error bars (than the data points towards the middle of Figure 7) or the data points towards the top of the graph lie closer to the line of best fit than in Figure 7.

Using the gradients of these graphs from the Matlab package and the thermometer measured temperatures; the ratio of *e/k* could be calculated. Then a weighted average had to be done since the six different sets of measurements (each with different combinations of resistors and water temperature) yielded different uncertainties. The gradients of these graphs corresponded to;

$$m = e/kT \qquad (1)$$

where T is the particular water temperature used and m is the gradient of that graph. Since e/k is constant it can be deduced from Equation 1 that;

$$m_H T_H = m_L T_L \qquad (2)$$

where m_H and T_H are the gradient and temperature respectively for a measurement at the boiling point of water whilst m_L and T_L correspond to the same but for a measurement at the freezing point of water. Note, this only works for a pair of measurements with the same resistor. Equation 2 can then be rearranged to;

$$T_L = \frac{(T_H - T_L)m_H}{m_L - m_H} \qquad (3)$$

The interpretation of Equation 3 is that a value for 0^0C can be obtained on the Kelvin scale. Using the fact that the Kelvin and Celsius scale differ by a constant figure of 273.15 a value for 0K can be obtained. Again, this produced three different values for 0K (from the three different resistors) with different uncertainties therefore a weighted average had to be carried out.

The main results are summarised in Table 1 below.

Ratio of elementary charge to Boltzmann's constant *e/k*	10848 ± 11 KV^{-1}
Absolute zero on Celsius scale	-264.8 ± 1.2 ^{0}C

Table 1. Summary of main results.

All uncertainties in Table 1 are quoted to two significant figures and the value is then quoted to the appropriate significant figure following this.

4. Discussion

The main factor contributing to the uncertainties in *e/k* and absolute zero was the fractional uncertainty in the gradient of the graphs. For example, the percentage uncertainty in the gradient of Figure (120L) was 0.22% (quoted to two significant figures). Combining errors in quadrature through the weighted averages led to a percentage uncertainty of 0.11% in *e/k* and 0.46% for absolute zero. The uncertainty in gradient is only a statistical uncertainty so the physical factor that contributed most to this uncertainty was the uncertainty in the circuit current.

The very small size of the error bars and high values of reduced-χ^2 for Figures 3 through to 8 means that the initial precision of the experimental apparatus may have been underestimated. But it is difficult to see where this was done as electrical equipment is usually very precise. It seems that the reduced-χ^2 increases with the value of resistance. This could be interpreted as meaning that the relation

$$I = I_0 e^{-\frac{eV}{kT}} \qquad (4)$$

(where I is the circuit current, I_0 is the current flowing to the low electron density side within the diode and V is the diode voltage) is not obeyed as closely the higher the resistance. Another trend that can be seen throughout Figures 3 to 8 is that at least the first data point (and sometimes data points towards the middle of the graph) lies further from the line of best fit than data points towards the top of the graph. A possible meaning of this is that Equation 4 is not followed as closely the lower the diode voltage. Displayed below are superposition graphs of results to show how closely Equation 4 is followed.

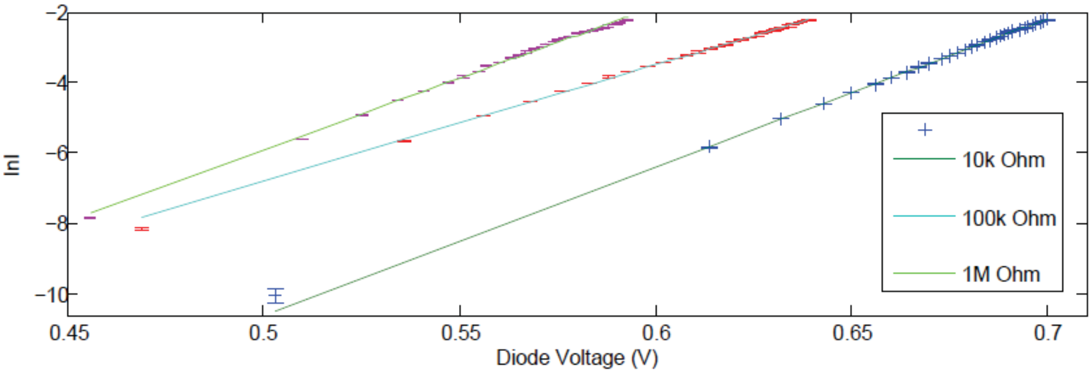

Fig 9. Superposition of Figure 3,5 and 7. Therefore it shows the low temperature measurements for the three resistors. 1MΩ is outermost to the left, then 100kΩ, then 10kΩ outermost to the right.

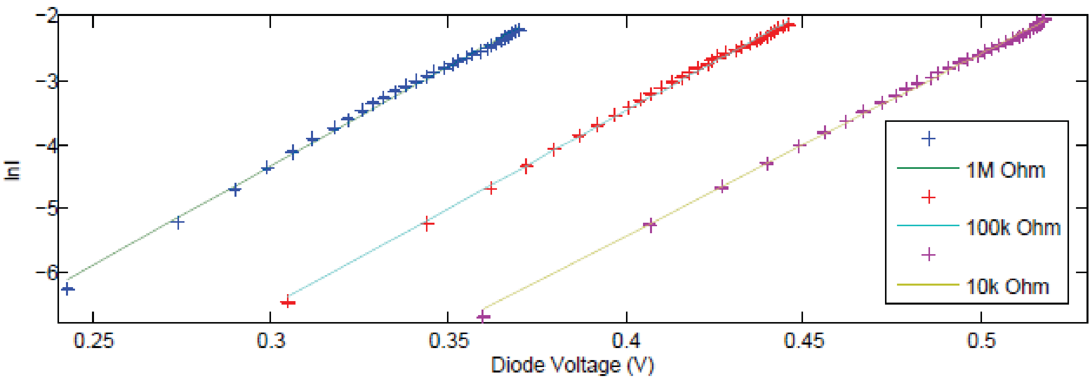

Fig 10. Similar to Figure 9, but for Figures 4,6 and 8. Therefore it shows the high temperature measurements for the three resistors. The order of graphs is the same as that for Figure 9.

As a result of Equation 1 it can be seen that the lines for the same temperature measurements with the different resistors should be parallel. In Figure 9 for low temperatures it seems this is approximately true for the measurements with the 10kΩ and 1MΩ resistors but not for the 100kΩ resistor. By eye it looks as if the 100kΩ graph would be parallel to the other two graphs if the first few measurements had a lower value for lnI, suggesting there was a mistake made here. However, Figure 10 for high temperatures seems to follow Equation 1 quite closely.

After analysing the data from our experimental results, it was found that the accepted value of 11605 KV^{-1} (quoted to the same amount of significant figures as the result obtained here) for *e/k* [3] does not lie within the uncertainties. In fact, the result is 66σ (quoted to two significant figures) out from the accepted value due to the very small final percentage uncertainty. The accepted value for absolute zero of -273.15 ^{0}C does not lie within the uncertainties either, but this time the result is only 6.8σ (again quoted to two significant figures) out.

5. Summary

The values obtained for the ratio of elementary charge to Boltzmann's constant and absolute zero were reasonable yet still far away from the accepted values when considered along with the final uncertainties obtained in the experiment. The small error bars in Figures 3 through to 8 leading to small percentage uncertainties in the final results suggest that there was an underestimate made in the precision of the original apparatus. The high values for reduced-χ^2 in Figures 3 through to 8 might mean that the semi-conducting diode did not follow the relation in Equation 4 as closely as was hoped.

References

[1] Beiser, Arthur, Concepts of Modern Physics, 6th Ed, McGraw-Hill, pp. 299.

[2] Matlab Least Squares Fit, lsfr26.m available from Teachweb, http://teachweb.ph.man.ac.uk/.

[3] Young, H.D. & Freedman, R.A. 2014, University Physics, 13th Ed, Pearson, pp. 665 & 771.

The number of words in this document is 1999.

This document was last saved on 3/2/2015 at 15:05