Measurement of the acceleration of gravity using Kater's pendulum

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This experiment was performed in collaboration with *John Cobbledick*.

Abstract

Through measurement of the periods of both ends of a Kater's pendulum using a stable frequency source, measurements of the acceleration due to gravity at Manchester were made. The pendulum consisted of a metal rod with two parallel knife-edges, defining the length, as well as the pivot and oscillating point. The result for the acceleration due to gravity was g = 10.23 ± 0.40 ms^{-2}. The main factor limiting the precision of the experiment was the measurement of the length.

1. Introduction

The period T of a simple pendulum of length l (for example a mass on a string) follows the relation:

$$T = 2\pi\sqrt{l/g} \qquad (1)$$

when oscillating. Equation 1 is based on a number of assumptions such as the string remaining taut and the mass of the string being negligible compared to the mass on the end of it. It is also hard to determine exactly where the centre of mass of such a pendulum is. Therefore, it is experimentally difficult to obtain an accurate value for the acceleration due to gravity by measuring the periods and lengths of different pendula.

A pendulum was built by Kater, designed to overcome this problem and accurately measure the acceleration due to gravity. The pendulum consists of two knife-edges, which means that one knife-edge can be oscillating whilst the other acts as pivot, and then their positions switched, just by flipping the pendulum upside-down. This gives two different periods for the pendulum. There are also two masses placed along the pendulum. Changing the position of a mass relative to the centre of mass of the system alters the two periods.

2. Theory

Using Newton's second law on the two different ends of the same pendulum, the moment of inertia about the centre of mass can be eliminated to produce the equation;

$$\frac{8\pi^2}{g} = \frac{T_1^2 + T_2^2}{a_1 + a_2} + \frac{T_1^2 - T_2^2}{a_1 - a_2}, \qquad (2)$$

where T_1 is the period from having knife-edge 1 oscillate, T_2 the period when knife-edge 2 is oscillating, a_1 is the distance of knife-edge 1 from the centre of mass and a_2 is the knife-edge 2 to centre of mass distance. The sum of distances $a_1 + a_2$ in Equation 2 can be measured very accurately using a travelling microscope, but the difference in distance $a_1 - a_2$ has to be measured using a ruler, introducing error into the measurements. To minimise the effect of the error in $a_1 - a_2$ in the final error in g the masses are moved until both periods T_1 and T_2 are brought as close as possible to each other, meaning that $T_1^2 - T_2^2$ is as small as possible. This is done so that the term including $a_1 - a_2$ will be small in the calculation of g (and therefore in the calculation of the error in g). However, it is not possible to have the knife-edges equidistant from the centre of mass as this would cause $a_1 - a_2$ to be 0.

3. Experimental method

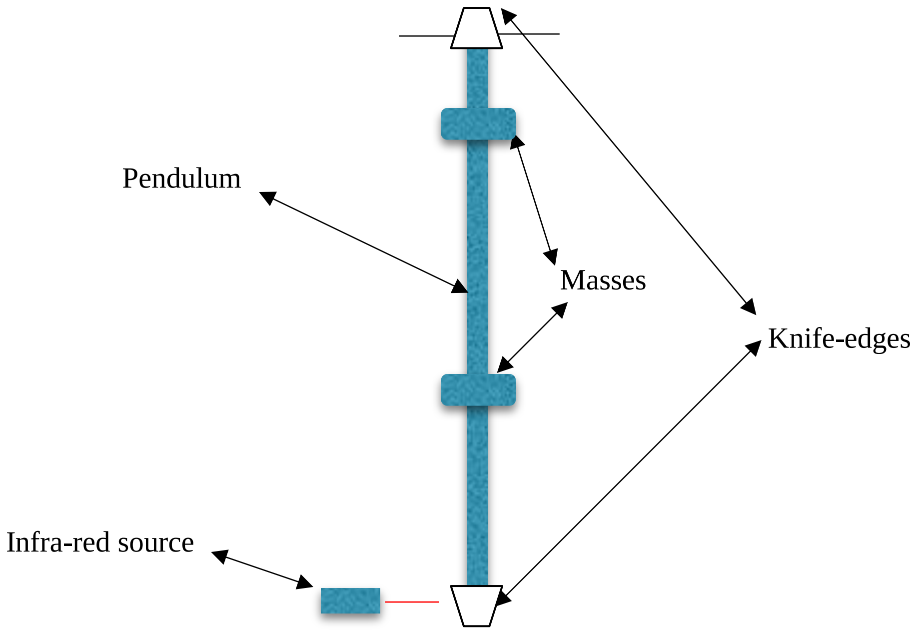

Fig 1. A simple apparatus diagram.

Periods were obtained using an infra-red light source connected to a counter that had an inbuilt 100kHz crystal oscillator. The infra-red light was always on but was only reflected by a bright material near the knife-edge; allowing the light to be reflected only when the knife-edge passed through the light. Measurements of twenty periods were made and the result divided by twenty to minimise the effect of releasing the pendulum compared to measuring one period. Since the pendula would continue oscillating for a huge number of periods, measurements were taken once they had stabilised by letting them swing for a number of minutes.

Masses were moved in a trial-and-error fashion to begin in an attempt to get T_1 and T_2 to coincide. Once a pattern (such as moving a particular mass up would decrease T_2) was found this pattern could either be followed or opposed to try get T_1 and T_2 closer to each other. Measurements of both periods were made until they agreed to within four decimal places of each other.

Once this was done, the pendulum was moved to a travelling microscope to measure the distance between the knife-edges, which is the same as the sum of the distances the knife-edges and centre of mass (term $a_1 + a_2$ in Equation 2 - referred to as l). Afterwards, the pendulum was placed on another knife-edge which lay on a table to find the centre of mass. Again, this had to be done using a trial-and-error method similar to when measuring the periods. Then the distances from knife-edge to centre of mass were measured using a ruler, whilst the pendulum was still balancing.

4. Results

The measured values can be found in Table 1;

Period 1 - T_1 (s)	1.88739 ± 0.04198
Period 2 - T_2 (s)	1.88741 ± 0.00703
Length of pendulum - l (cm)	92.357 ± 0.001
Distance from knife-edge 1 to centre of mass - a_1 (cm)	29.1 ± 0.1
Distance from knife-edge 2 to centre of mass - a_2 (cm)	63.0 ± 0.1

Table 1. Summary of main results.

For the period measurements, the results are quoted to the number of significant figures that shows that they are different. The error on the period measurements is given by the difference between the period when it coincided with the other and the period measured on the next experiment day (having not changed anything).

Shown are the measurements for T_1 and T_2 until they coincided;

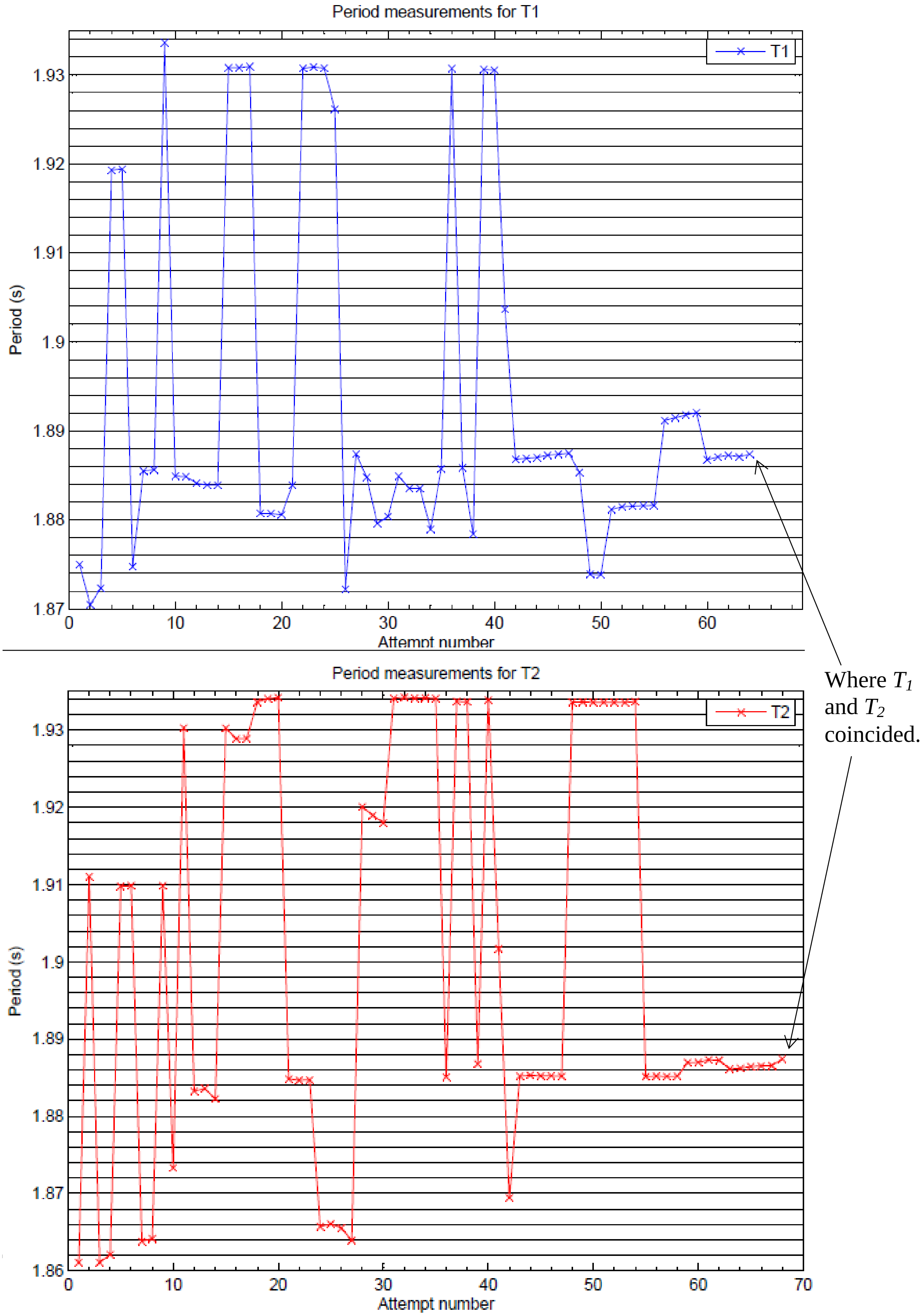

Fig 2. A graph of the measurements of T_1 and T_2 until they coincided, produced using Matlab [1].

5. Discussion

There are a number of factors which have to be taken into account when trying to obtain an accurate value for the acceleration due to gravity. These include; thermal expansion of the metal from which the pendulum is made, stretching of the pendulum due to the masses, buoyancy of the pendulum in air, finite radius knife-edges and the latitude and height of the experiment location.

Where possible, an estimate for the correction required due to these various factors was made. Sometimes this was not possible as it would have required extra measurements with unavailable high precision equipment. Precise room temperature measurements were not taken and therefore it was not possible to make a good estimate of the thermal expansion, it was assumed it stayed at room temperature throughout the experiment and therefore did not see a measurable temperature change. Measurements of air and the pendulum's material densities as well the volumes of the masses were not possible with the available apparatus, denying the possibility of making a correction for buoyancy.

It can be seen from Equation 3 [2];

$$\Delta g = -3.086 \times 10^{-6} h \qquad (3)$$

where Δg is the correction in the calculated value of g and h is the height above sea level, that the correction due to the height of the location is negligible. With a typical height of 3m for a workbench on the first floor of a building the ratio $\Delta g/g$ is of order 10^{-6}. A paper [3], which conducted the same experiment as this and was able to make some of the high precision measurements, quoted the correction due to stretching to be -0.004 ms^{-2} (making value of g slightly lower than quoted value). After including this correction, a new value of g = 10.23 ± 0.40 ms^{-2} was obtained.

6. Summary

With Manchester at a latitude of 53.30^0 [4], the accepted value for the local acceleration due to gravity can be calculated using Equation 4 [2];

$$g = 9.780327(1 + 0.0053024\sin^2\varphi - 0.0000058\sin^2 2\varphi) \qquad (4)$$

where φ is the latitude angle. This gives a result of g = 9.813 ms^{-2}, leaving it slightly more than 1σ from the value of g = 10.23 ± 0.40 ms^{-2} obtained in this experiment. The final value for the acceleration due to gravity is higher than the accepted value since the periods changed so easily. It was found that measuring T_1, then T_2, then T_1 again without changing the position of either mass would change T_1, sometimes quite drastically! This might mean that the masses were slipping, without having been fully tightened.

References

[1] MATLAB, Version 5, The Math Works Inc, Natick, Mass 01760.

[2] http://www.sensorsone.com/local-gravity-calculator/ 15:53 14/4/15

[3] Jeffreys, H. FRS, On The Absolute Measurement Of Gravity, 19/3/1949

[4] http://www.infoplease.com/ipa/A0001769.html 14:06 1/4/15

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