Thermal diffusivity of plastic

John Cobbledick and Meirin Evans

9437432 and 9214122

School of Physics and Astronomy

The University of Manchester

Second Year Laboratory Report

Nov 2015

Abstract

Through measurement of the axial temperature of a sample of epoxy resin using a thermocouple connected to a mechanical graph plotter with two separate methods of a sudden change in temperature and periodic changes, the thermal diffusivity of the sample was calculated. The result for the thermal diffusivity of the sample was 0.0900 ± 0.0003 mm^{2}s^{-1}. The main contributors to the error in thermal diffusivity were temperature measurements.

1. Introduction

The thermal diffusivity of a material is related to the heat flow through it. In this experiment, the material used was an epoxy resin (plastic) sample. The sample was an isotropic (same in all directions) [1] uniform solid in the form of a cylinder, with a thermocouple inside to measure its axial temperature. When the surface temperature of the sample is changed, the axial temperature changes as a result of the sample trying to attain thermal equilibrium with its surroundings. Two methods can be used to measure the thermal diffusivity of the sample: the first being to suddenly change its surface temperature and the other being to periodically change its surface temperature.

2. Theory

Case A - sudden change in surface temperature

When the surface temperature $\theta(a, t)$ (where a is the radius of the cylinder and t is the time since changing the surface temperature from a lower temperature θ_1 to a higher temperature θ_2 of the sample is suddenly changed (as shown in Figure 1), the axial temperature $\theta(0, t)$ will exponentially approach θ_2 after the transients effects have passed (as shown in Figure 2).

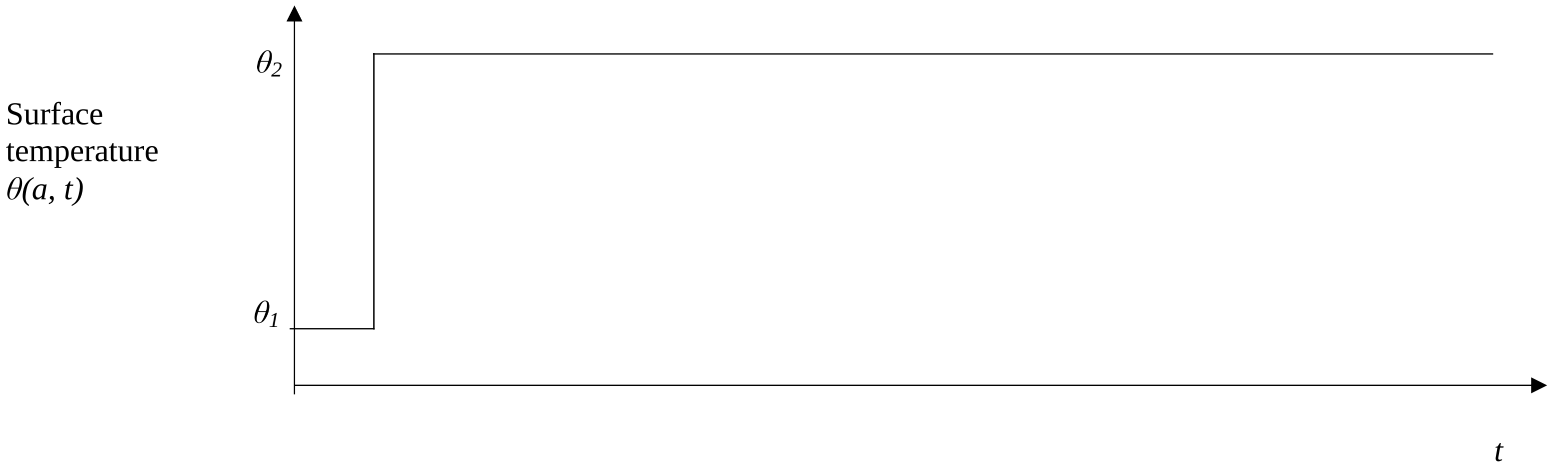

Fig 1. A simple graph of surface temperature against time for Case A.

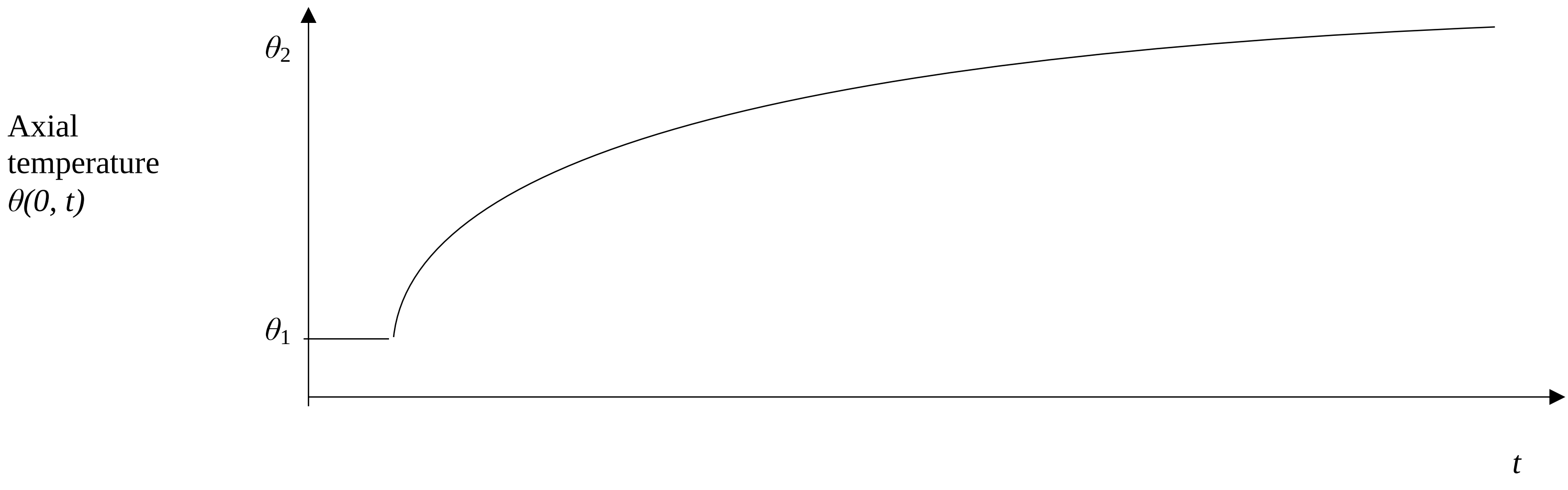

Fig 2. A simple graph of axial temperature against time for Case A.

This leads to the proportionality;

$$\frac{|\theta - \theta_2|}{\Delta\theta} \propto e^{-\lambda_1^2 Dt/a^2}, \qquad (1)$$

where $\Delta\theta = \theta_2 - \theta_1$, D is the thermal diffusivity and λ_1 is a dimensionless constant with value of 2.405. Plotting a graph of $ln(\theta-\theta_1)$ against t gives a gradient, m, of;

$$m = \frac{-\lambda_1^2 D}{a^2}. \qquad (2)$$

Equation 2 can be rearranged to give a value for D. For a sudden change in the sample's surface temperature from θ_2 to θ_1, the θ_2 in Equation 1 is changed to θ_1.

Case B - periodic change in surface temperature

When $\theta(a, t)$ is periodically changed between θ_2 and θ_1 (as shown in Figure 3), a heat wave is generated. This heat wave is damped between the surface and centre of the sample. This wave can be represented as a Fourier series. For short enough periods, T, the Fourier coefficients for n greater than 1 will be relatively small, leaving sinusoidal behaviour (as shown in Figure 4).

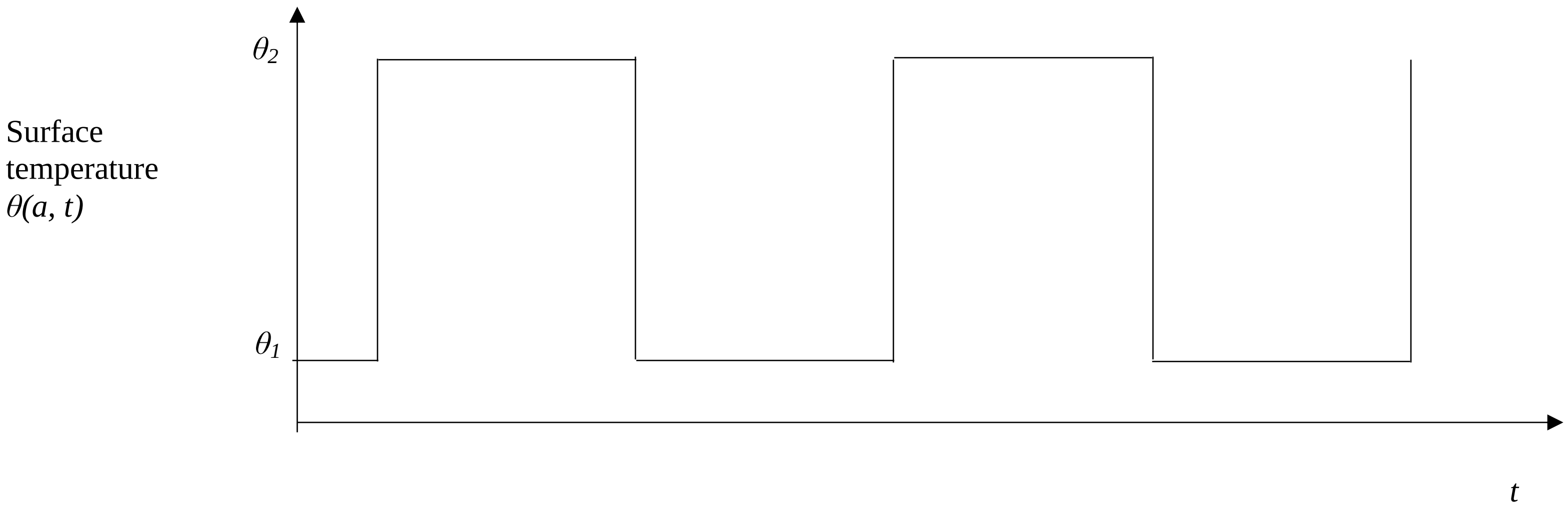

Fig 3. A simple graph of surface temperature against time for case B.

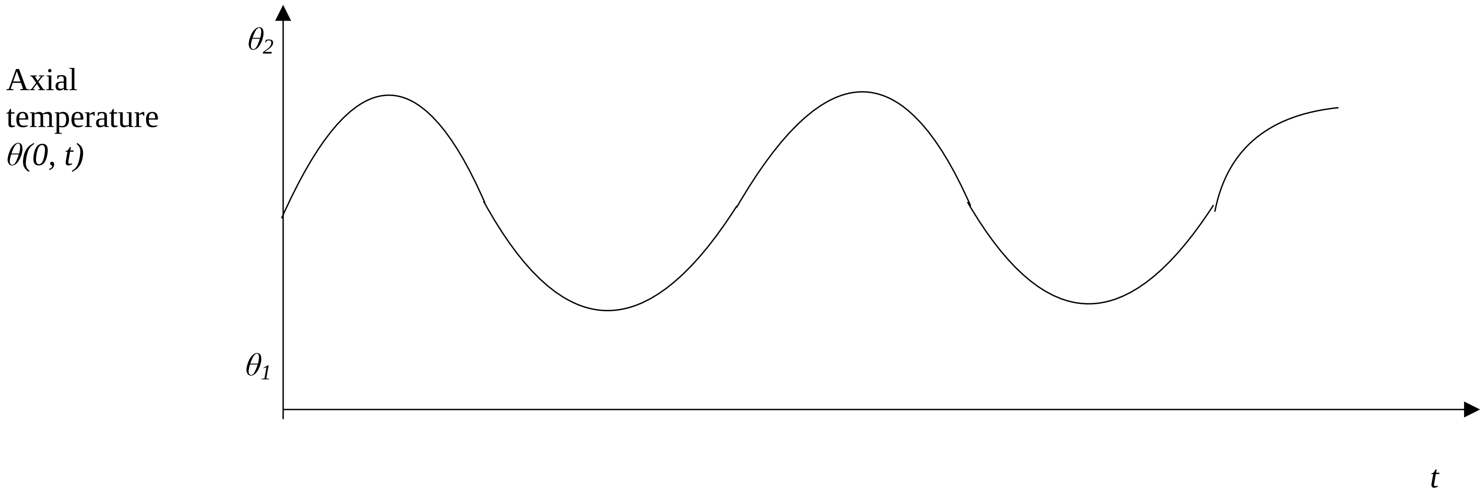

Fig 4. A simple graph of axial temperature against time for Case B. The axial temperature doesn't reach θ_2 and θ_1.

The peak-to-peak, *B*, of the sinusoidal wave in Figure 4 is related to *D* through Equation 3;

$$B = \frac{4(\theta_2 - \theta_1)}{\pi \left| M_0 (a\sqrt{\frac{2\pi}{TD}}) \right|}, \qquad (3)$$

where M_0 is the Kelvin function.

There is a phase lag, ϕ, between changing $\theta(a, t)$ and the peaking of $\theta(0, t)$, which is related to D through Equation 4;

$$\phi = \arg M_0 (a\sqrt{\frac{2\pi}{TD}}) \qquad (4)$$

3. Experimental procedure and set-up

Shown in Figure 5 is an apparatus diagram which was used for both cases A and B.

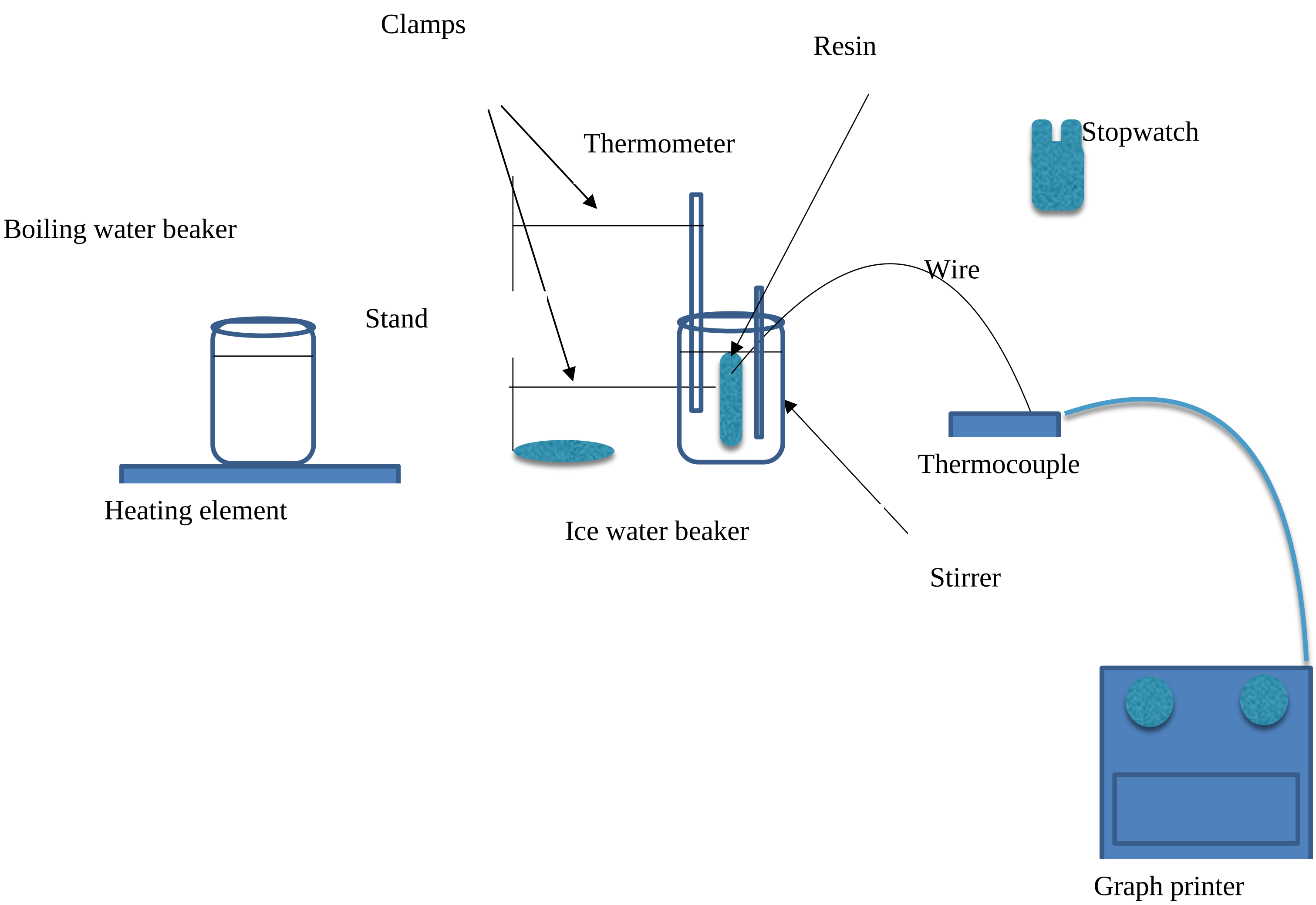

Fig 5. A simple diagram of the apparatus used during the experiment.

A beaker full of ice water was used to give a θ_1 of 0^0C. In the same way, a beaker of boiling water with the heating element on full power was used to give a θ_2 of 100^0C. A stirrer was used to keep $\theta(a, t)$ at 0^0C and a thermometer to check that the water was at the correct temperature. Also, the resin had to be completely submerged in the water to ensure an accurate $\theta(a, t)$. A thermocouple (digital thermometer) was connected to the inside of the resin, and therefore measured $\theta(0, t)$. This thermocouple was connected to a graph plotter that outputted the exponential curve for case A and sinusoidal curve for case B. A stopwatch was needed to measure t for case A and T for case B.

4. Results

Case-A

Using Equation 2, a straight line fit using the Matlab Least Squares Fit package was produced for the resin being immersed in boiling to freezing water and again for the opposite situation [2].

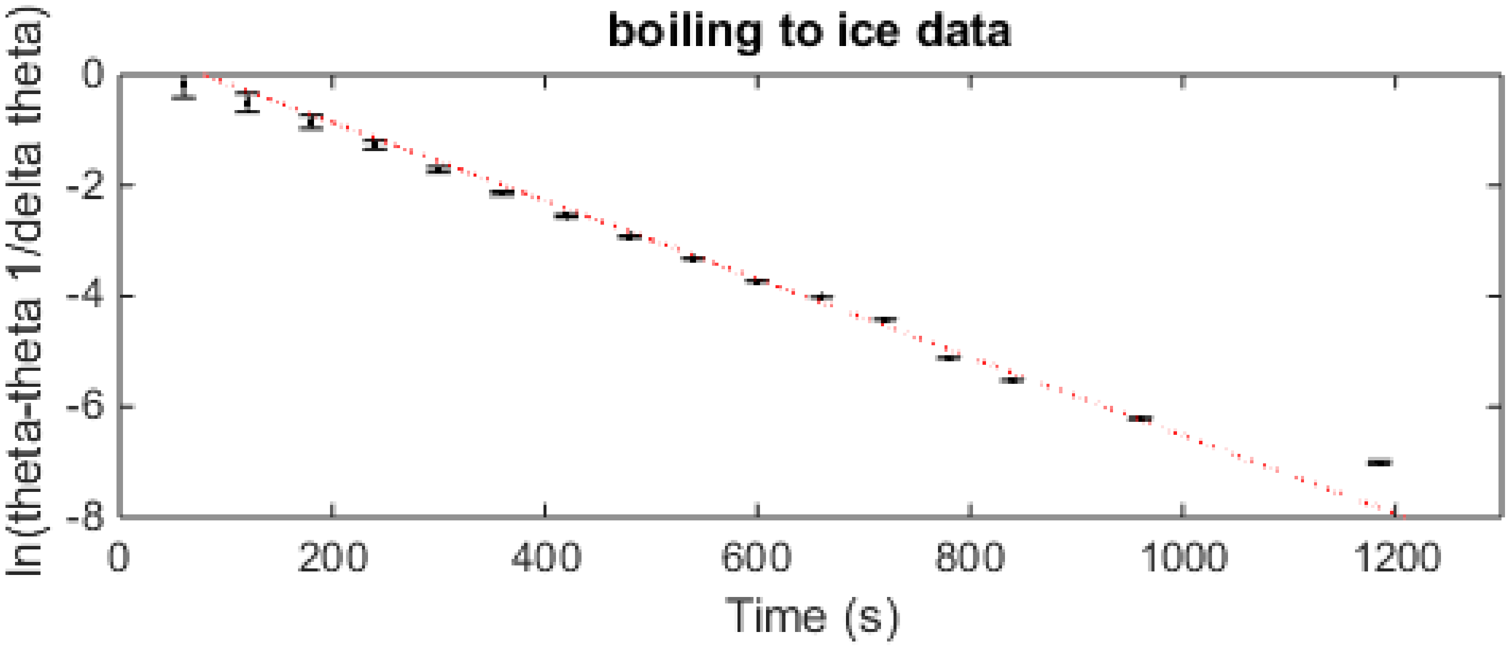

Fig 6. A plot of $\frac{ln(\theta-\theta 1)}{\Delta\theta}$ on y axis against *t* on x axis. There were two fitted variables which were the gradient and the y intercept. The *m* value was -0.00708 ± 0.00002 s^{-1}.

The reduced-χ^2 obtained by fitting the data from Figure 6 was 3235 which is an extremely poor fit. There are large uncertainties on the initial points as well as a final point which does not lie close to the fitted line. However, the majority of the central data points do lie on the fitted line, suggesting that a linear fit is suitable but that errors on the individual points must be revised.

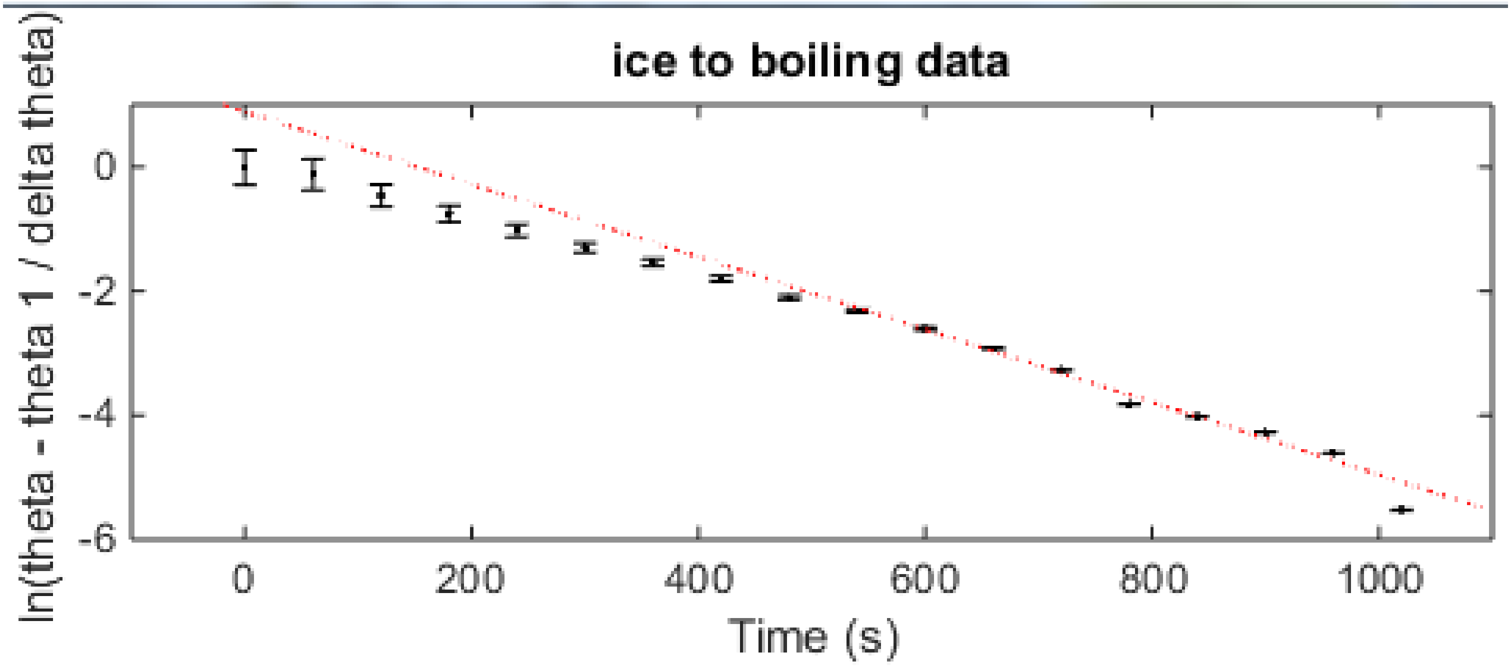

Fig 7. Same fit parameters as Figure 6 but with more data points. The *m* value was -0.00584 ± 0.00002 s^{-1}.

The reduced-χ^2 obtained by fitting the data from Figure 7 was 6074, which comparing with the reduced-χ^2 from Figure 6, suggests an even poorer fit. Many of the points, particularly at the beginning, have high levels of error and have the largest residuals hence affecting the value of the reduced-χ^2.

Equation 2 allowed D to be calculated. Errors on D were propagated in quadrature involving partial derivatives. The dominant error which arose from this error analysis was in the subtraction of θ-θ_1.

The values obtained for D with their associated errors are summarized in Table 1. A repeat for each transition from hot to cold temperatures was made. However, when a repeat was made for the transition from hot to cold, the data produced an exponential curve which made a linear fit unsuitable, hence this data was discarded.

Case-B

With case B, two estimates of D were found using the same data but by looking at two different properties of the produced graphs. Data for periods of 2, 3, 4, 5 and 9 minutes respectively were taken. These data produced sinusoidal curves, after initial effects disappeared. Measurements of the peak to peak values from several periods were made, from which a weighted average was taken. By rearranging Equation 3 for M_0, an estimate for its argument could be determined by data provided from a table of the Kelvin function [3]. The data from the table were used to plot the Kelvin function as shown in Figure 8.

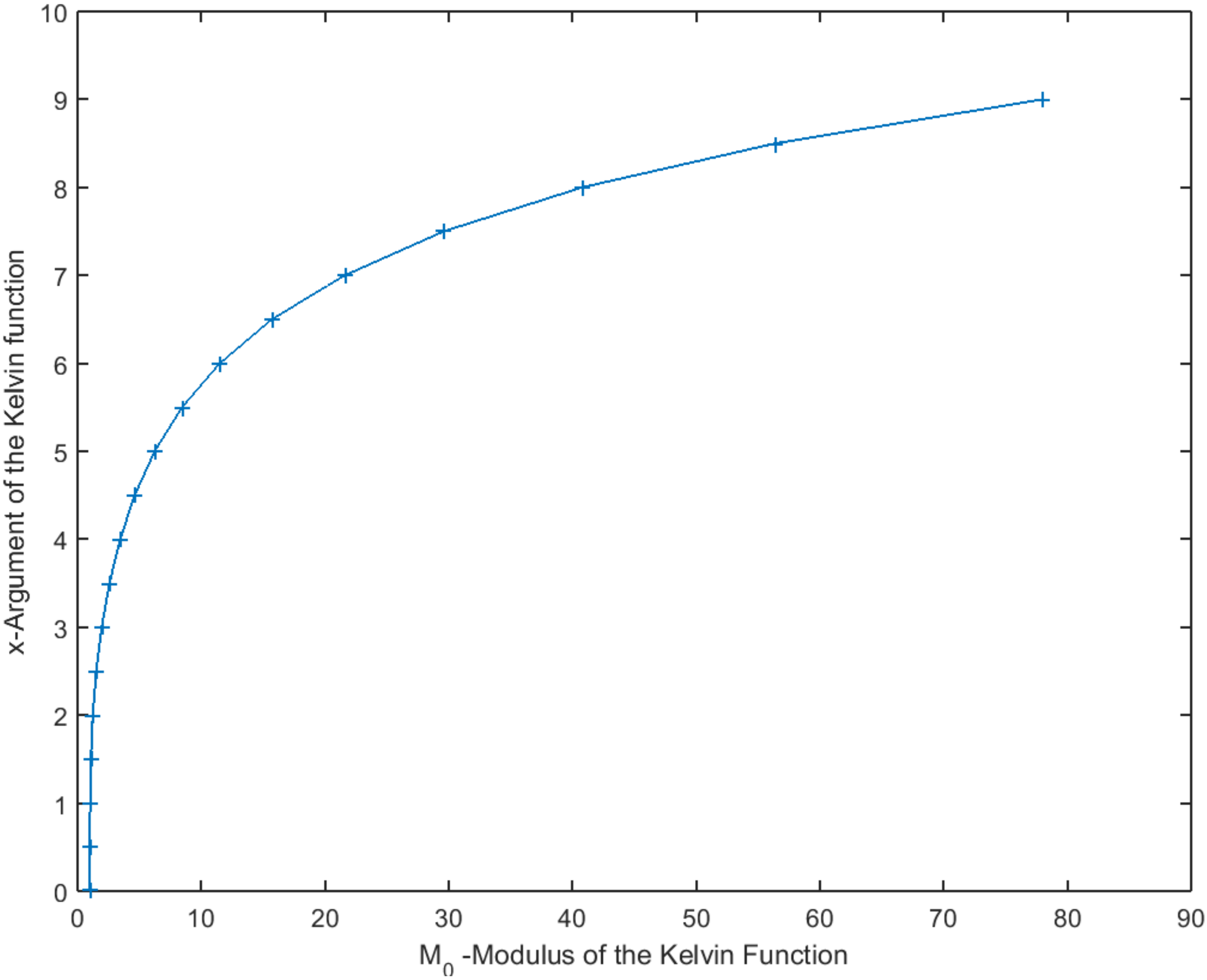

Fig 8. The Kelvin function.

Each of the points in Figure 8 have been connected by linear interpolation, this made estimating error due to interpolation difficult. Error in M_0 was obtained using quadrature from Equation 4, and $M_0 + \sigma$ and $M_0 - \sigma$ were plotted in order to obtain a range for the argument. An example of this is shown in Figure 9. The error of the argument was therefore assumed to be twice the largest difference between M_0 and $M_0 + \sigma$ or M_0 and $M_0 - \sigma$. An estimate on the uncertainty of T was made based on contributions from human reaction time and the time taken to move the resin to water of a different temperature. Thus we obtained an uncertainty of $\sigma_T = \pm 3s$. Once these uncertainties were estimated, an estimate of the uncertainty of D could be made through quadrature, with results shown in Table 2. Figures 10 to 13 show graphs for each T.

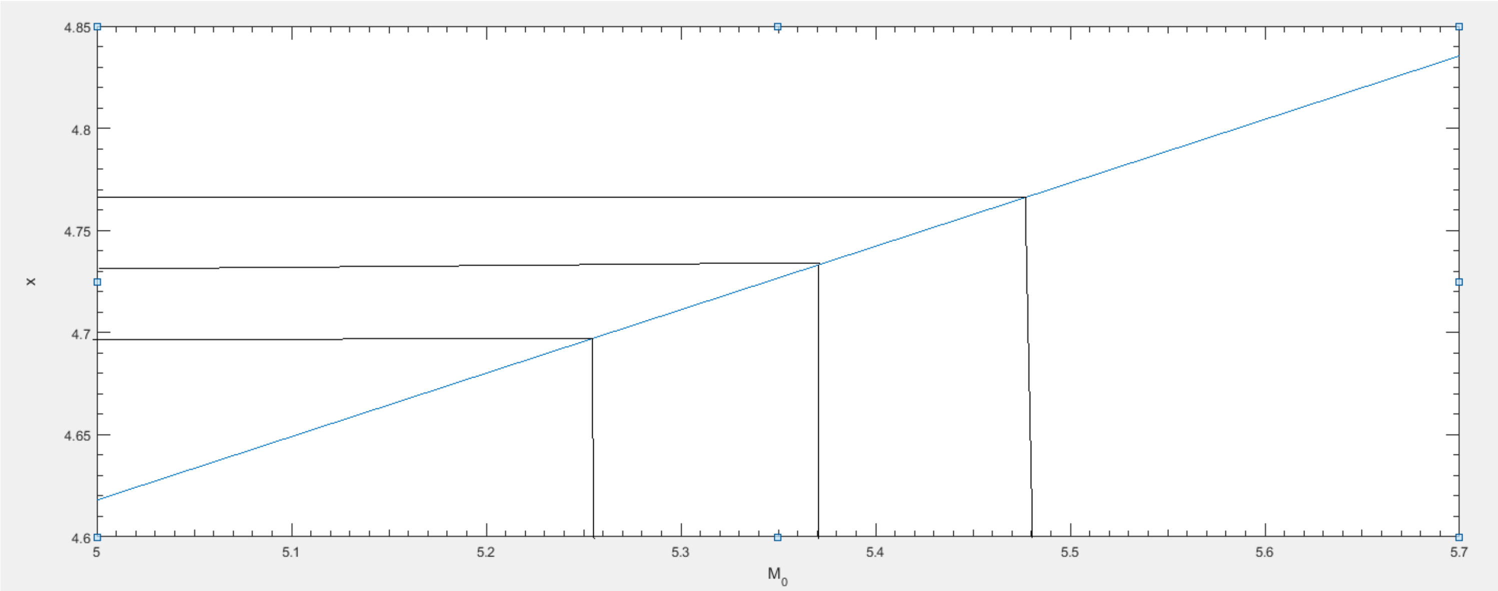

Fig 9. Kelvin function for a period of 4 minutes. Central estimate is the value obtained by M_0 and the lower and upper estimates are from $M_0 - \sigma$ and $M_0 + \sigma$ respectively.

In Figure 10, a smooth sinusoidal pattern is clearly shown, with the maxima and minima similar for successive cycles. Similar graphs were observed for T of 4 and 5 minutes.

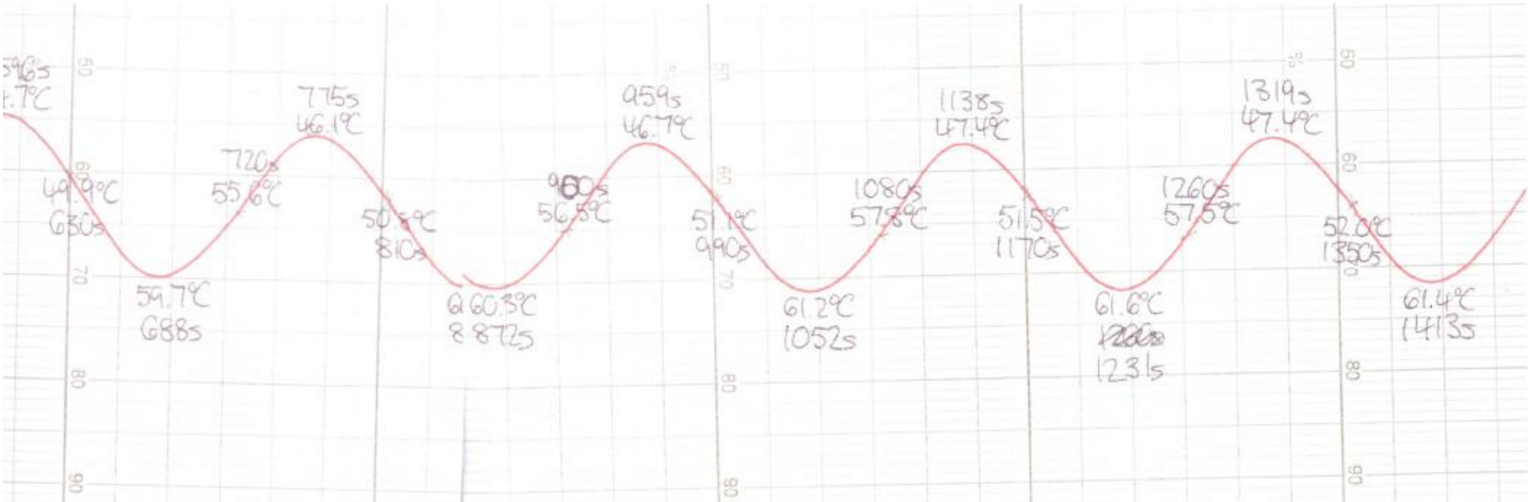

Fig 10. Graph for case B with T = 3 minutes. Graphing machine speed at 1 cm/min. Further periods are omitted due to length of the graph.

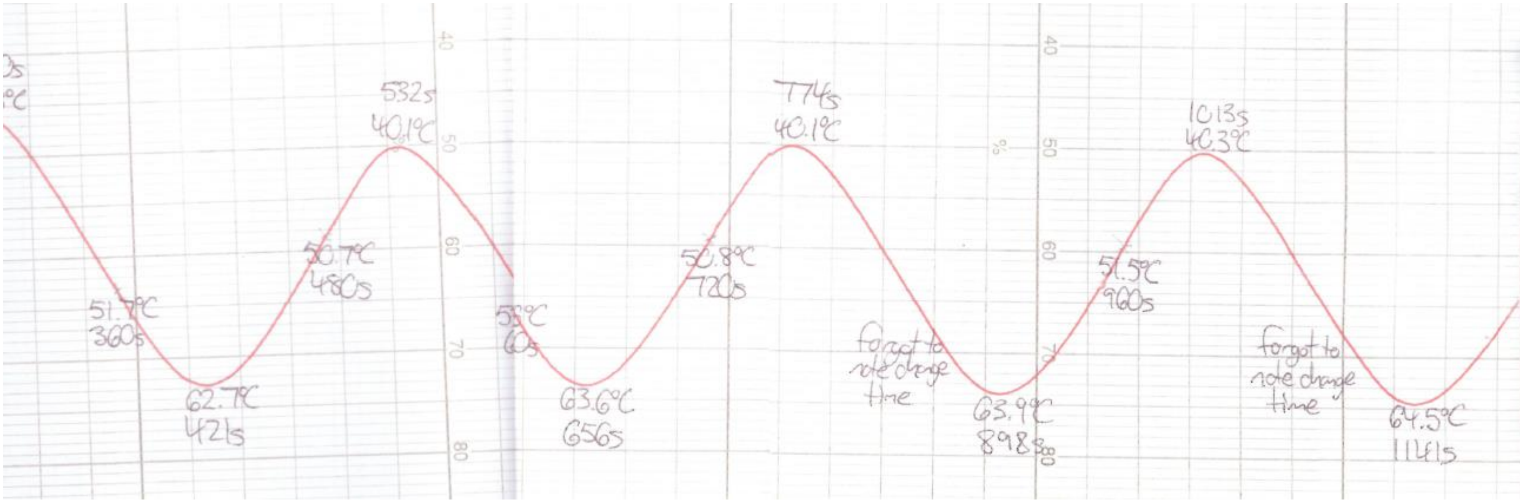

Fig 11. Graph for case B with T = 4 minutes. Graphing machine speed at 1 cm/min. Further periods are omitted due to length of the graph.

A T of 2 minutes produced an unstable sine curve, as shown in Figure 12, and this T was deemed too short. In Figure 12 there is visible irregularity between the maxima and minima of each cycle.

Fig 12. Graph for case B with T = 2 minutes. Graphing machine speed at 1 cm/min.

With Figure 13, although the maxima and minima were similar between cycles, there was significant curving when approaching maxima or minima, which produced a non-sinusoidal curve.

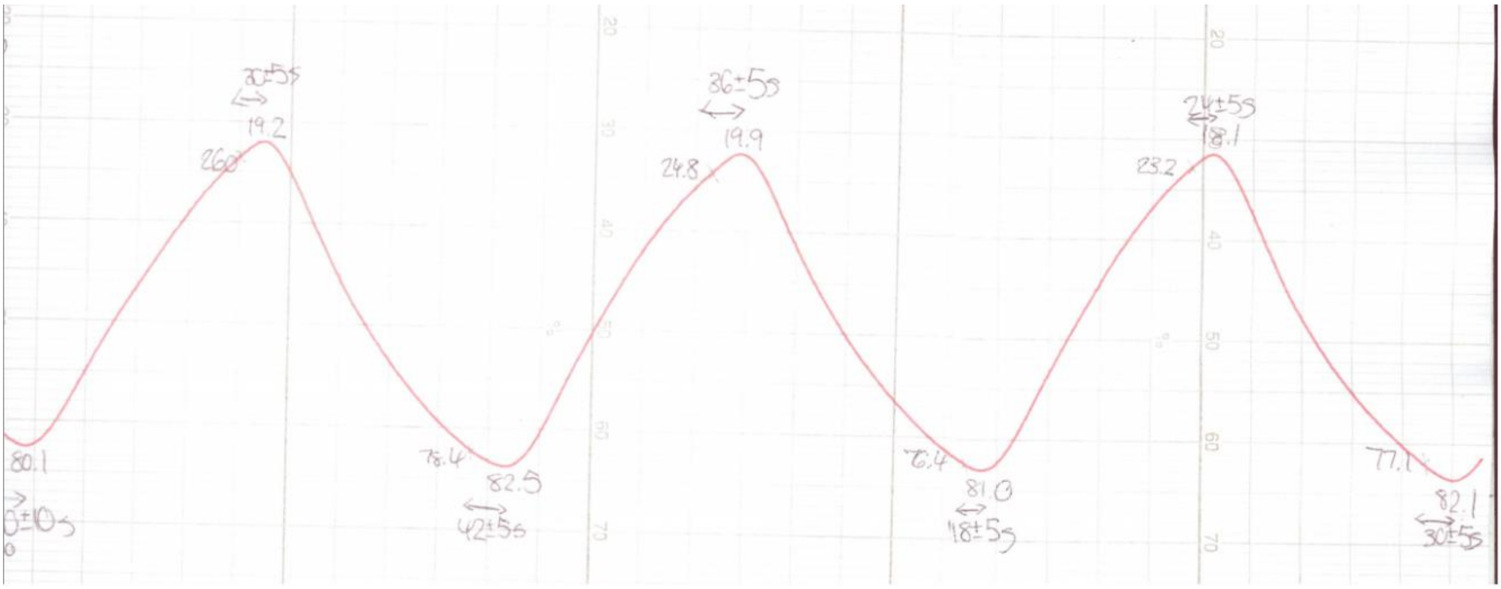

Fig 13. Graph for case B with T = 9 minutes. Graphing machine speed at 1 cm/min.

Despite the non-sinusoidal nature of Figure 13, an estimate of D was made using the peak to peak values from the data in order to provide a comparison between the values obtained from data using other periods. An estimate using the phase lag for this period was not made due to the non-sinusoidal nature.

As for estimating D using phase lag, it is unclear how the argument of M_0 behaves, therefore the method for calculating M_0 from peak to peak could not be used. Instead, calculated values for ϕ were compared with the table for M_0, giving an inequality in D. The average of the two sides of the inequality was taken as the D value, with the uncertainty quoted as half the difference between the two extremes of the inequality. The uncertainties in D values from this method were high as it was not possible to interpolate the argument of M_0 graphically. The results for this method are shown in Table 3.

Transition	D (mm^{2}s^{-1})
Boiling to freezing	0.0929 ± 0.0006
Freezing to boiling	0.077 ± 0.001
Freezing to boiling repeat two	0.074 ± 0.001

Table 1. Summary of main results for case A.

Period *T(minutes)*	D(mm^{2}s^{-1})
3	0.085 ± 0.002
4	0.089 ± 0.001
5	0.093 ± 0.001
9	0.093 ± 0.001

Table 2. Summary of main results from case B for an estimate of D using peak to peak data.

Period T (Minutes)	D (mm^2s^{-1})
3	0.030 ± 0.004
4	0.03 ± 0.05
5	0.33 ± 0.07

Table 3. Summary of main results from case B for an estimate of D using phase lag data.

Using these values, weighted averages were taken in order to produce an average value for each experiment and an overall average, these results are summarized in Table 4.

Experiment	Average D(mm^2s^{-1})
A	0.0878 ± 0.0005
B (peak to peak)	0.0920 ± 0.0005
B (phase lag)	0.06 ± 0.04
A and B (without phase lag)	0.0896 ± 0.0005
A and B	0.0900 ± 0.0003

Table 4. Weighted averages for each case and combined.

5. Discussion

The results have uncertainties of order of 1-2%. The confidence in these uncertainties is low. Firstly, considering the factors in uncertainty in case A, there was uncertainty in the m, temperature and a. Due to the precision in measuring a using calipers, this uncertainty is insignificant. However, the uncertainty in m depends explicitly on the uncertainty of the temperature, which itself arises from the thermocouple. The large reduced-χ^2 for both transitions suggests that the fitted line was poor and that the uncertainty in the temperature is much larger than estimated.

This larger uncertainty in temperature also contributes to the uncertainty in case B when using peak to peak data, due to explicit dependence on uncertainty in temperature. It should be noted, however, that weighted averages from both cases agree to the same order of magnitude and are within 1 to 2 standard deviations of each other. Other major contributors to uncertainty for values from peak to peak data were the uncertainty in the argument of M_0 and T. Uncertainty from the argument was approximately 1-1.5% for each T but an uncertainty of 6.25% was calculated for T = 3 minutes. The percentage uncertainty in T also increases as T decreases with a maximum uncertainty of 1.7% for T of 3 minutes. If further time were available, more cycles would be measured in order to estimate a more consistent value for peak to peak and therefore consistent values for uncertainty of the argument. Additional repeats for case A would provide an additional value for the transition from boiling to freezing, since a repeat produced unexpected exponential behavior.

6. Conclusion

Through the use of 3 methods, several values of D were obtained. A weighted average from case A and case B (peak to peak) produced an estimated D of $0.0900 \pm 0.0003\ \mathrm{mm^2 s^{-1}}$. The conclusions reached from experimental results were that the value of D is of the order $1 \times 10^{-8}\ \mathrm{m^2 s^{-1}}$ and that the main contribution to the error in D arose from uncertainty in temperature.

References

[1] Zemansky, M.W. & Dittman, R.H. Heat and Thermodynamics, McGraw-Hill, 7th Ed, pp. 96.

[2] Matlab Least Squares Fit, lsfr26.m available from Teachweb, http://teachweb.ph.man.ac.uk/.

[3] Abramowitz, M. & Stegun, I.A. Handbook of Mathematical Functions for Science Students, Dover, 10th Ed, pp.432

The number of words in this document is 1888.

This document was last saved on 23/11/2015 at 13:46